Progress Report

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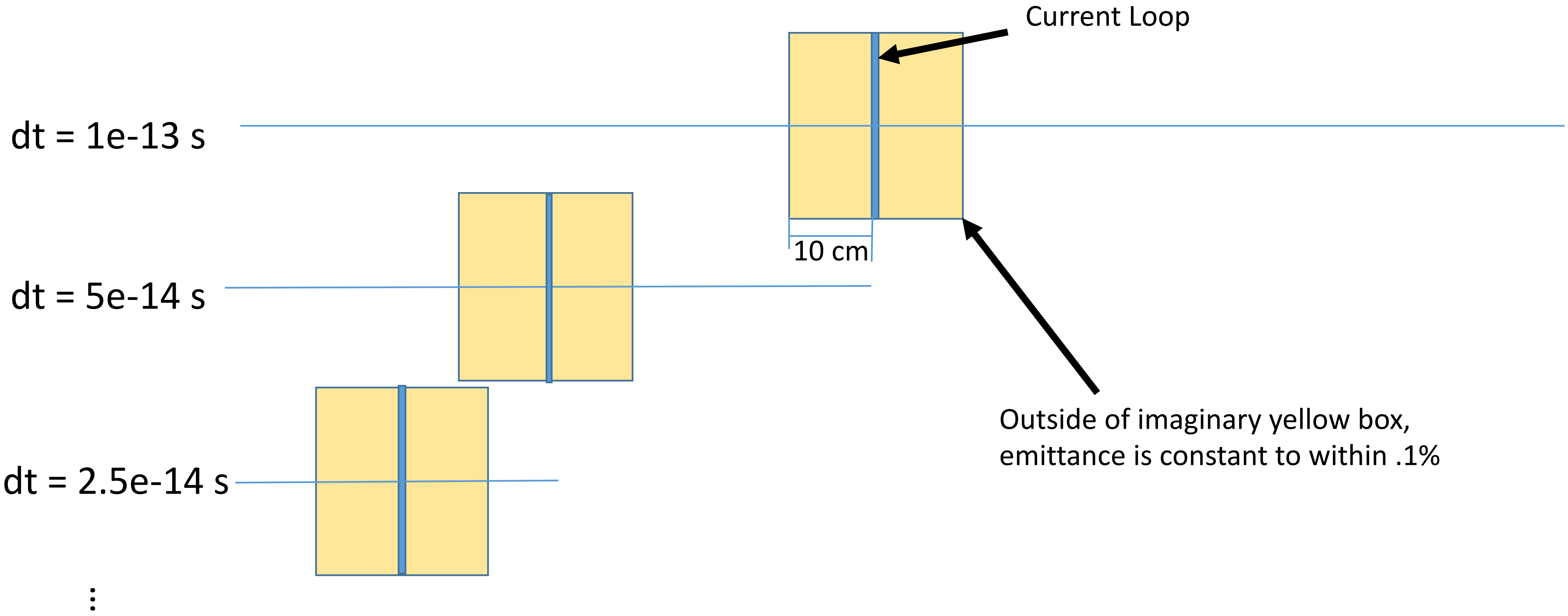
Geometric Aberration + Convergence Study

- Simple Test Case: Use emittance component tool to identify geometric aberration emittance growth from a current loop in simulation
- Test convergence of GPT emittance to integrated emittance component
- See comparison between theoretical emittance growth and simulation

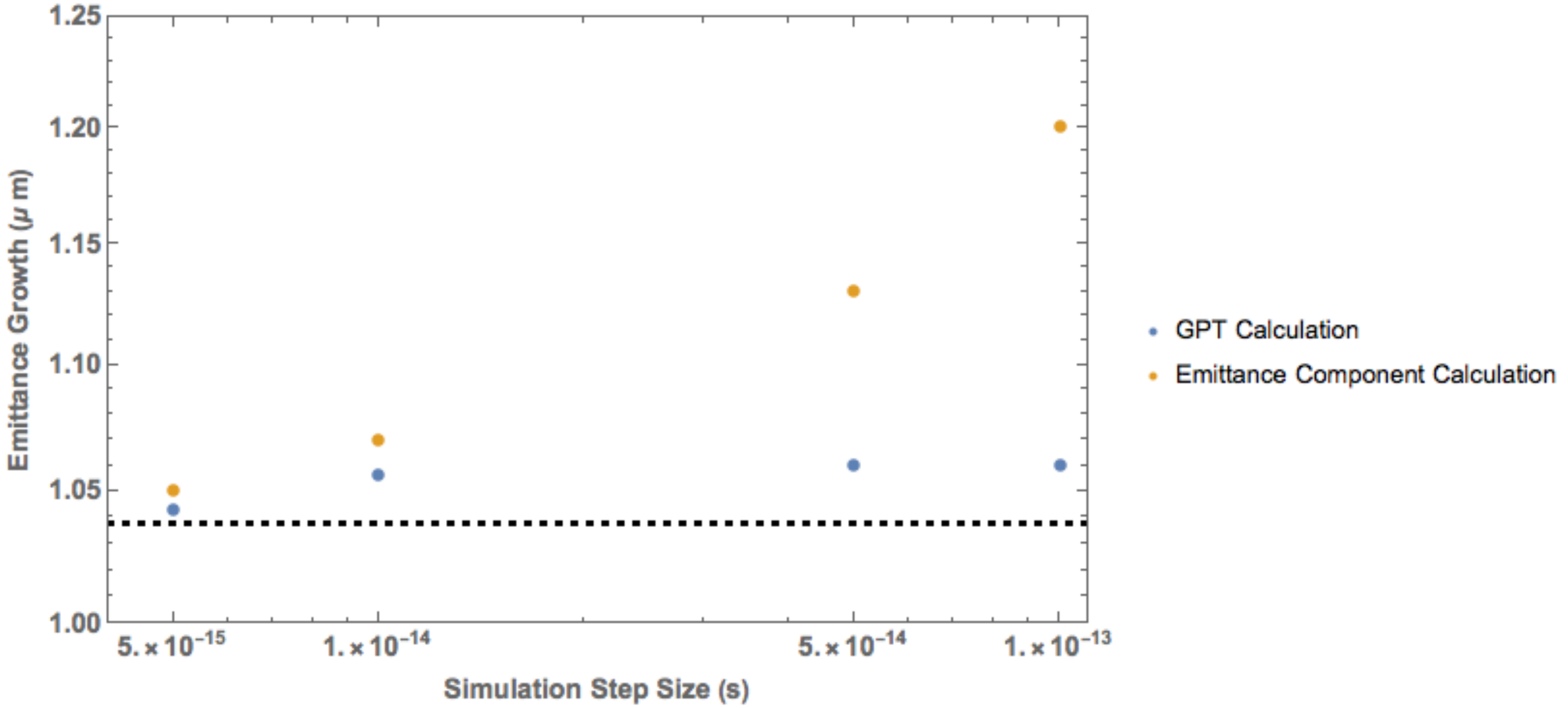
Convergence Test

- Time constraints on simulation runtime mean I can't simply decrease time step, I needed to decrease it in a way to ensure simulation time doesn't grow



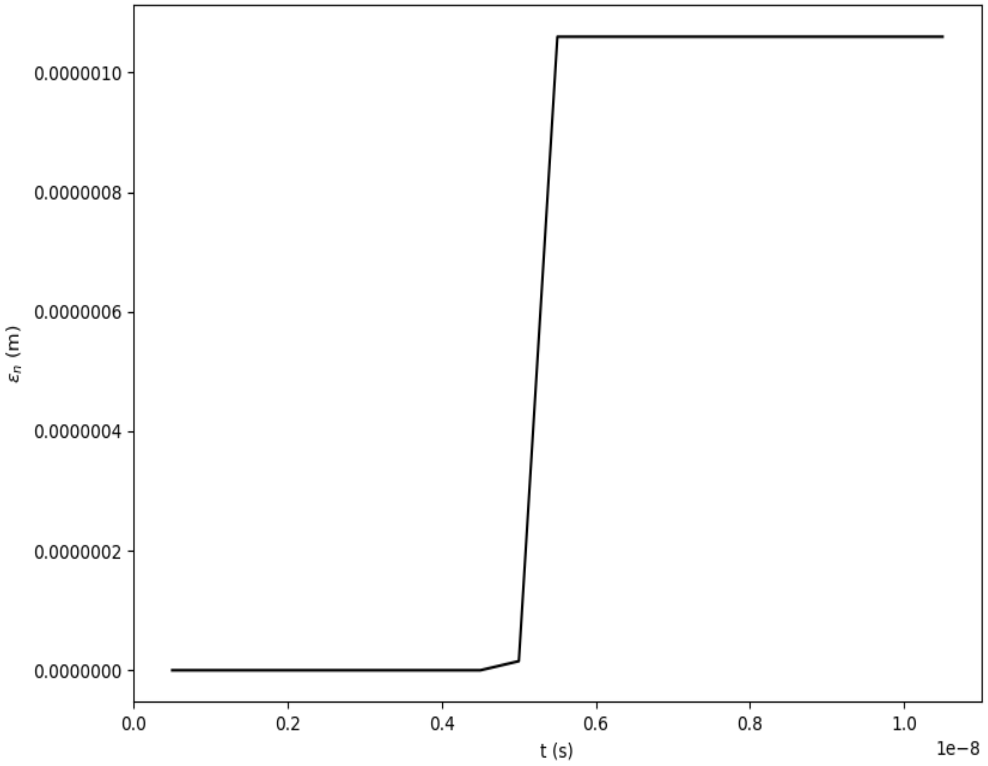
Comparing to Theoretical Geometric Aberration

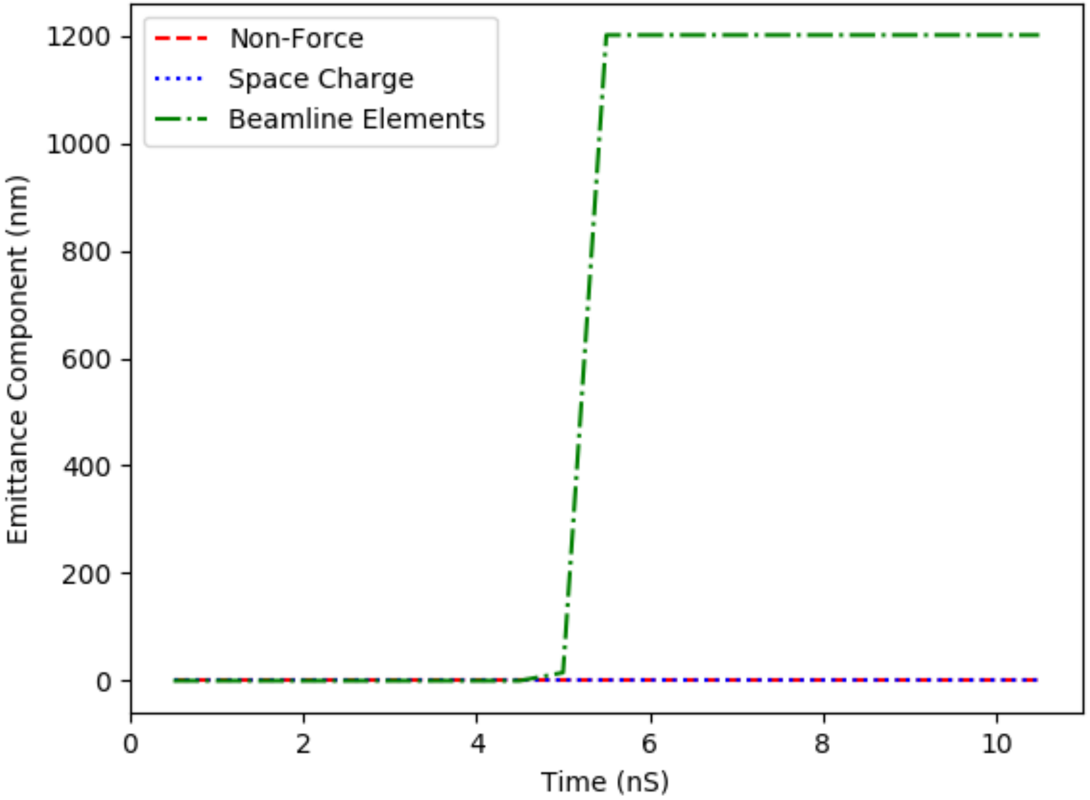
- GPT emittance growth corresponds to theoretical growth (1.037 μm) to .6%



GPT Emittance and Emittance Component

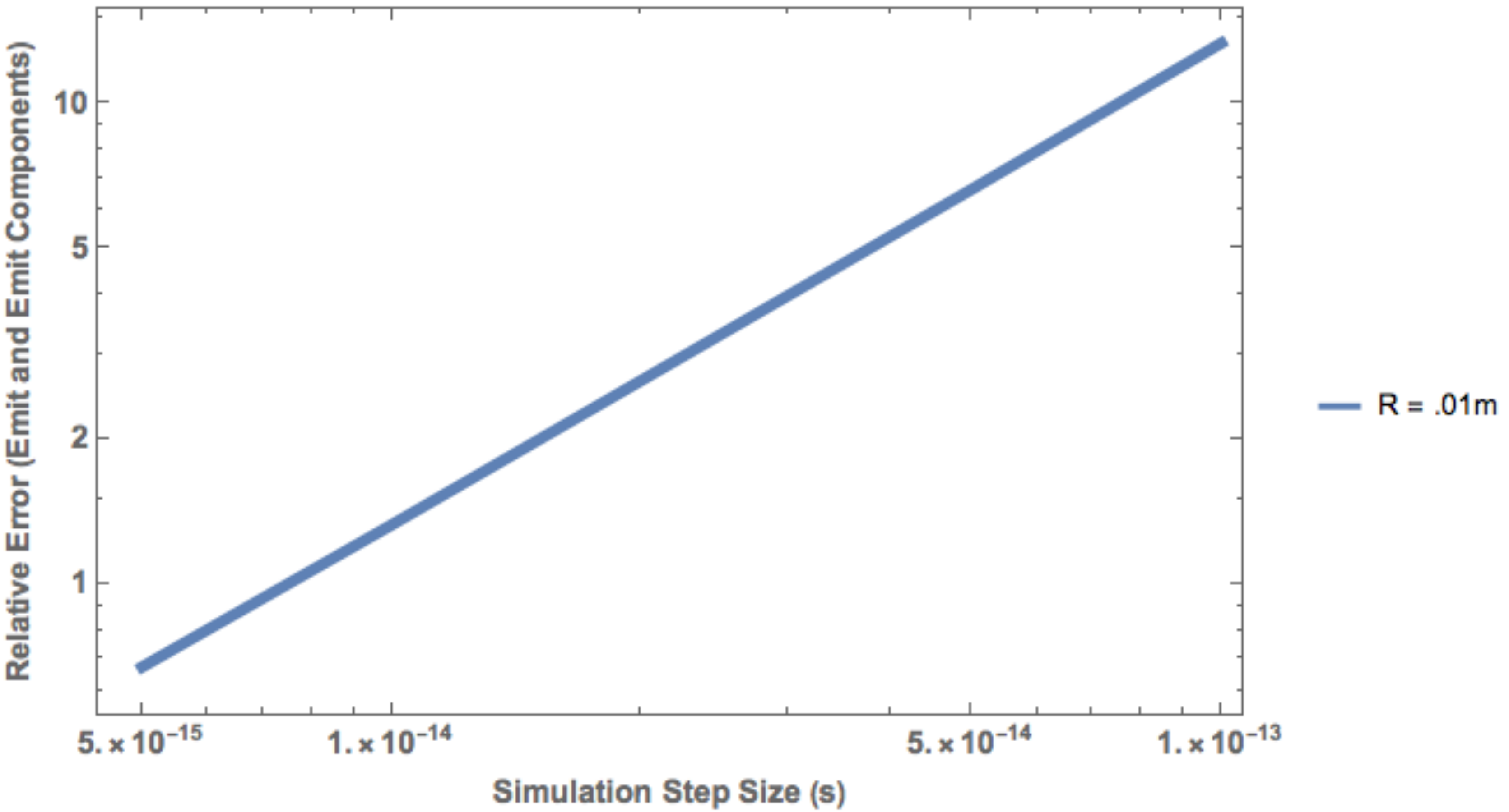
- Algorithm correctly attributes emittance growth to beamline elements





Convergence Test Results

- Emittance Component calculation converges to GPT value within 1% by 5e-15 s step size



Backup

```
# Set the GPT license
GPTLICENSE=1251405651;

# Define the beam parameters
gamma = 1.92;
radius = 1e-3;
length = 1e-6;
number_of_particles = 1000;
total_charge = -10E-15;
#total_charge = 0;
initial_emittance = 1e-10;

#Radius and Current of Current Loop
#R=.01;
I=50000;
#Divisor used to scale timestep and lattice
#DVR = 10;

# Start the beam
setparticles("beam", number_of_particles, me, qe, total_charge);
setrxydist("beam", "u", radius/2, radius);
setphidist("beam", "u", 0, 2*pi);
setGdist("beam", "u", gamma, 0);
setzdist( "beam", "g", 0, length, 3, 3) ;

# Set its initial emittance to 1nm
setGBxdist("beam", "g", 0.0, 1e-3, 3.0, 3.0);
setGBxemittance("beam", initial_emittance);
setGBydist("beam", "g", 0.0, 1e-3, 3.0, 3.0);
setGByemittance("beam", initial_emittance);

#Current loop halfway through the beamline with radius R and Current I
solenoid("wcs","z",1.5/DVR*.857, R, I);

# Compute the emittance components
emittance_component("4d space charge","false");
emittance_component("4d beamline element","false");
emittance_component("4d non-force","false");

# Set the solver tolerance
dtmax = 1e-13/DVR;
accuracy(6);

# Print touts
tout(0.0, 1.05e-8/DVR, 5e-9/DVR);
```

4.7.13 Solenoid

`solenoid(ECS,R,I) ;`

Single turn solenoid.

ECS	Element Coordinate System.
R	Radius of the solenoid [m]
I	Current through the solenoid [A].

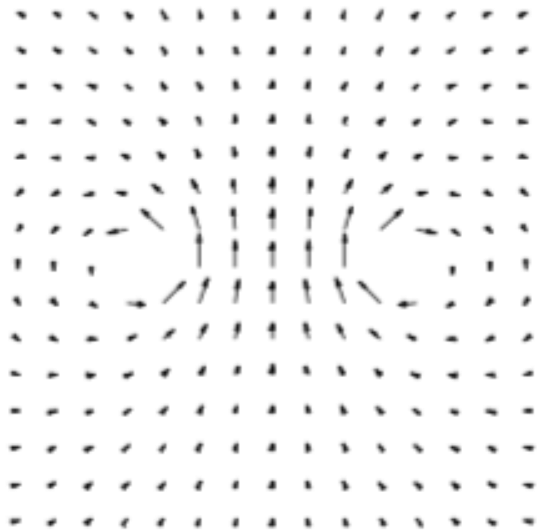


Figure 4-14: Magnetic field in the xz-plane.

The solenoid is modeled as a circle centered around the origin in the xy-plane with radius **R** carrying a current **I**. The resulting magnetic field is directed in the positive z-direction, within the current loop. The vector potential of the magnetic field is given in spherical coordinates by [16F22, pp. 117]:

$$A_\phi(r,\theta) = \frac{\mu_0 IR}{4\pi}\int_0^{2\pi} \frac{\cos(\phi)}{\sqrt{R^2 + r^2 - 2Rr\sin(\theta)\cos(\phi)}}d\phi \qquad [4.68]$$

The magnetic field is calculated from $\mathbf{B} = \nabla \times \mathbf{A}$. The integral and rotation are calculated analytically.

As the field is defined everywhere and no approximations are made, this element is very useful to study aberrations in solenoid lens systems.

Spherical Aberration Calculation

- Using simple current loop, calculating emittance growth due to spherical aberration from:
  Kumar, Vinit & Phadte, Deepraj & Bhai Patidar, Chirag. (2011). A simple formula for emittance growth due to spherical aberration in a solenoid lens.

- The emittance growth of a azimuthally symmetric beam due to a solenoid in the thin lens approximation is simply related to the geometry of the solenoid and the beam

$$\varepsilon_{xy} = \frac{R^4}{2\sqrt{6}\, f_0}\sqrt{\frac{C_1^2}{12} + \frac{C_1 C_2}{5} R^2 + \frac{C_2^2}{8} R^4}$$

- Where C1 and C2 are reductions of the focal length due to the 3rd and 5th order spherical aberrations respectively

$$C_1 = \frac{1}{2}\frac{\int_{-\infty}^{+\infty}\{B'(z)\}^2\,dz}{\int_{-\infty}^{+\infty}B^2(z)\,dz}, \quad C_2 = \frac{5}{64}\frac{\int_{-\infty}^{+\infty}\{B''(z)\}^2\,dz}{\int_{-\infty}^{+\infty}B^2(z)\,dz}.$$