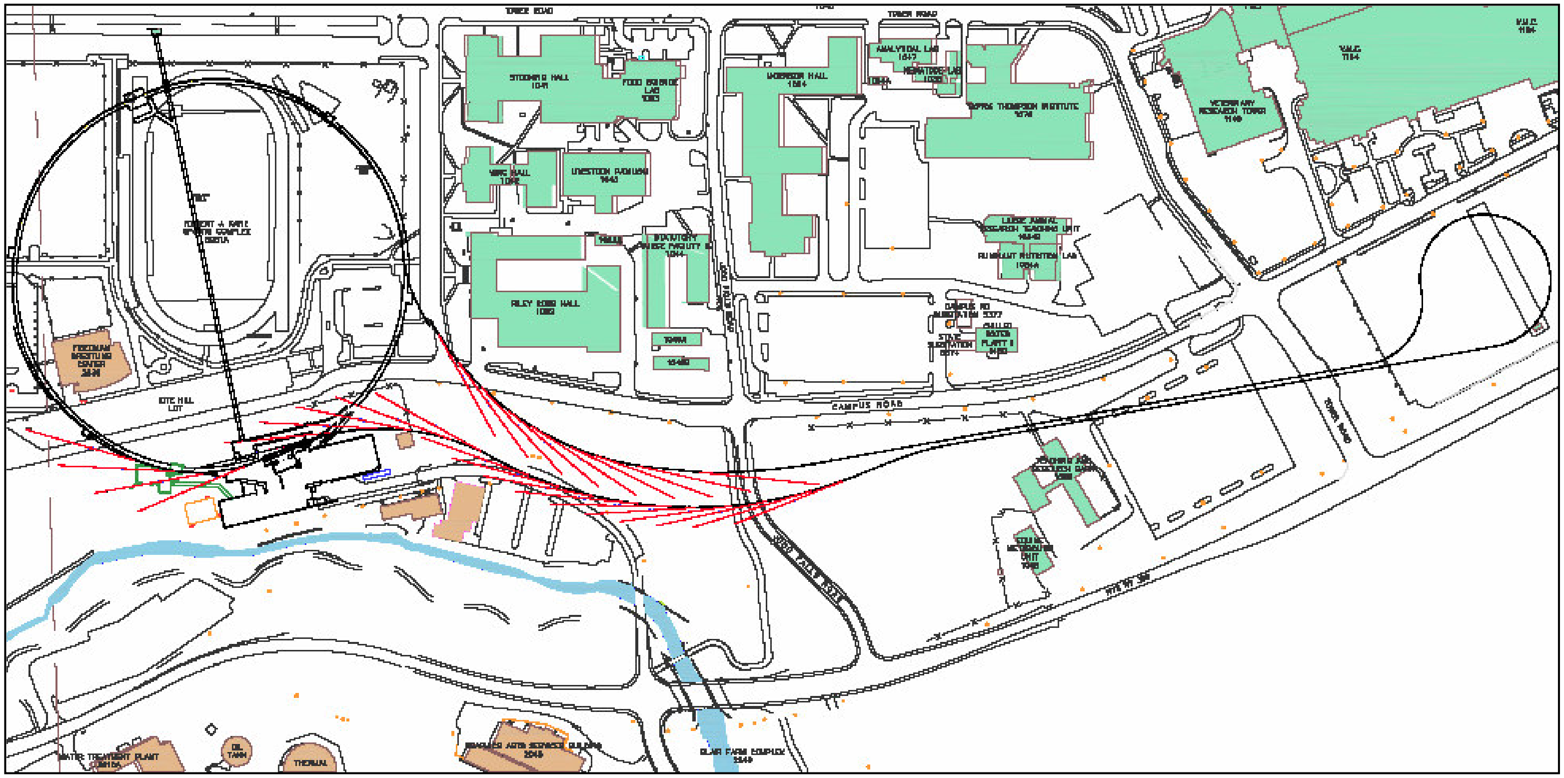
Beam Breakup Instability in the Cornell 5GeV ERL



Changsheng Song

Georg Hoffstaetter

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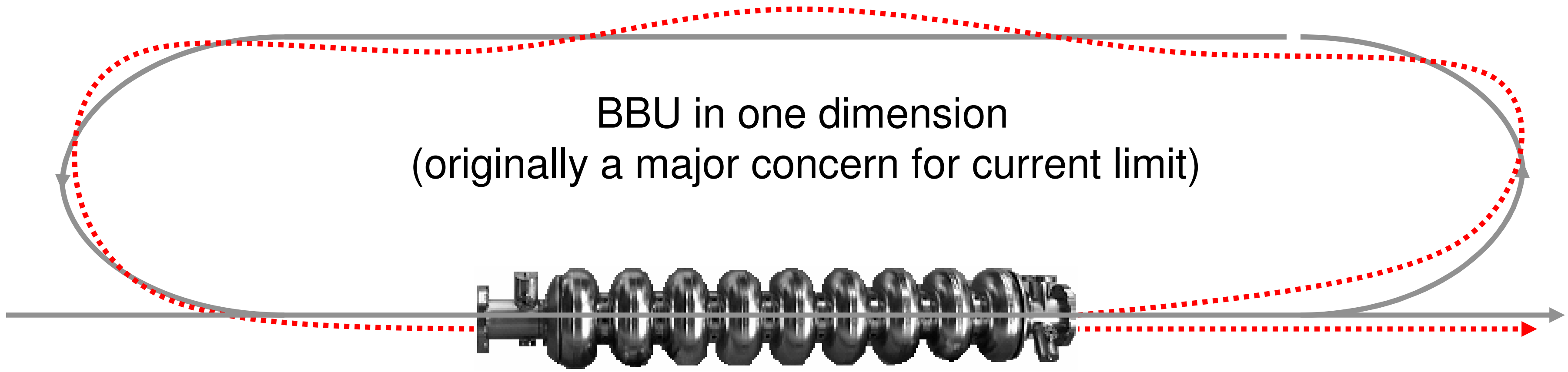
BBU Instability Overview



Three Major Types of BBU Instability

1. Monopole BBU Instability
2. Dipole BBU Instability
3. Quadrupole BBU Instability

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Monopole BBU Instability

The Four Most Dominant Monopole HOMs

	f_λ(**GHz**)	Q_λ	$(R/Q)_\lambda[\Omega]$
1	3.85763	13728	31
2	2.45658	1778.8	134.5
3	5.93396	27887	5.99
4	3.85758	40172	2.94

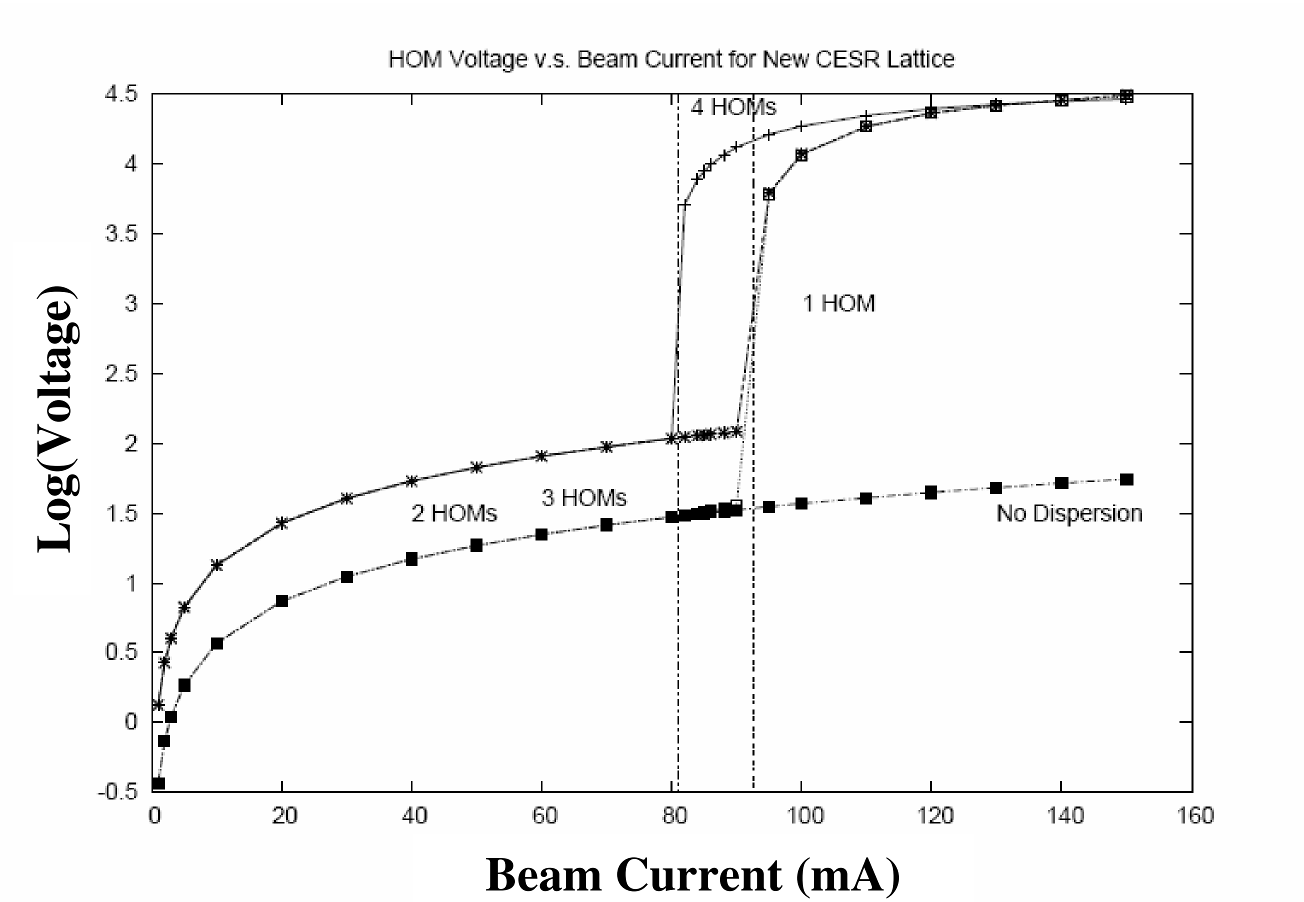
Approximate Formula for a Single Monopole HOM

$$I_{\text{th}} = \frac{2\beta c E_0}{r_{56}\omega_\lambda (R/Q)_\lambda Q_\lambda}$$

r56 : Time of Flight

Monopole HOM Power

- Monopole HOM Voltage vs. Beam Current



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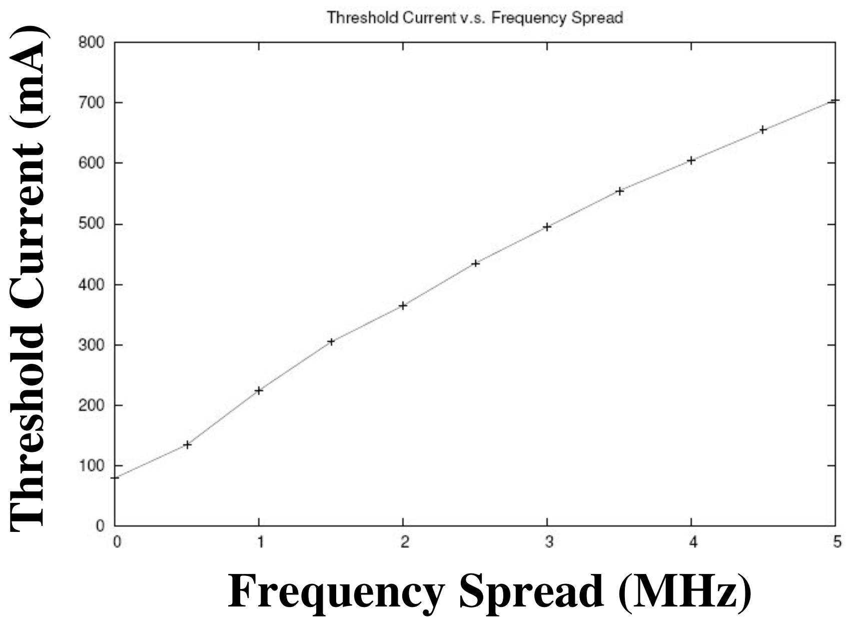
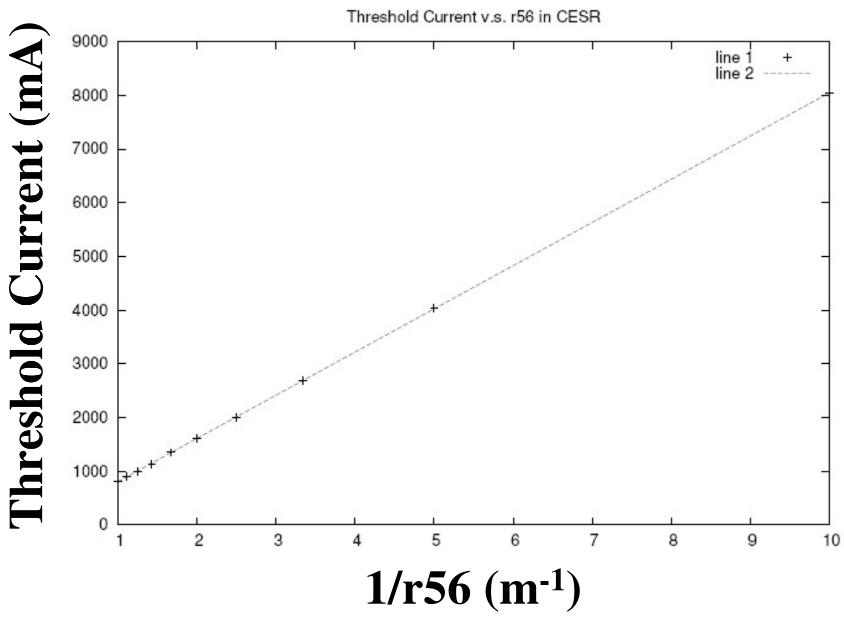
Monopole BBU Threshold Current

Achieving Higher Monopole BBU Threshold Current

The effect of HOM frequency randomization on the threshold current



The effect of the time of flight term r56 on the threshold current.



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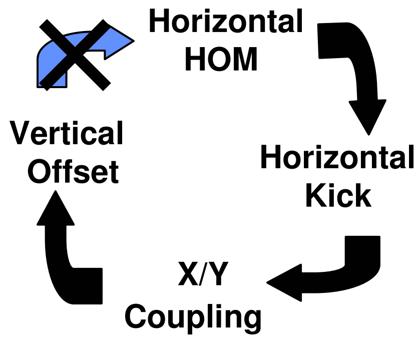
Dipole BBU Instability

Table 2: The eight dominant polarized transverse HOMs for the 7-cell ERL cavity.

	f_λ[GHz]	Q_λ	$(R/Q)_\lambda[\Omega]$	θ_λ
1	1.87394	20912.4	109.60	0
2	1.81394	20912.4	109.60	$\pi/2$
3	1.88173	13186.1	27.85	0
4	1.82173	13186.1	27.85	$\pi/2$
5	1.86137	4967.8	71.59	0
6	1.80137	4967.8	71.59	$\pi/2$
7	2.57966	1434.2	108.13	0
8	2.51966	1434.2	108.13	$\pi/2$



Approximate Formula for a Single Dipole HOM

$$I_{\rm th} = -\frac{2c^2}{e\left(\frac{R}{Q}\right)_\lambda Q_\lambda \omega_\lambda} \frac{1}{T_{12}^* \sin \omega_\lambda t_r}$$

$$T_{12}^* = T_{12}\cos^2\theta_\lambda + \frac{T_{14}+T_{32}}{2}\sin 2\theta_\lambda + T_{34}\sin^2\theta_\lambda$$

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Threshold Current of Dipole BBU Instability

Threshold Current for Unpolarized Dipole HOMs

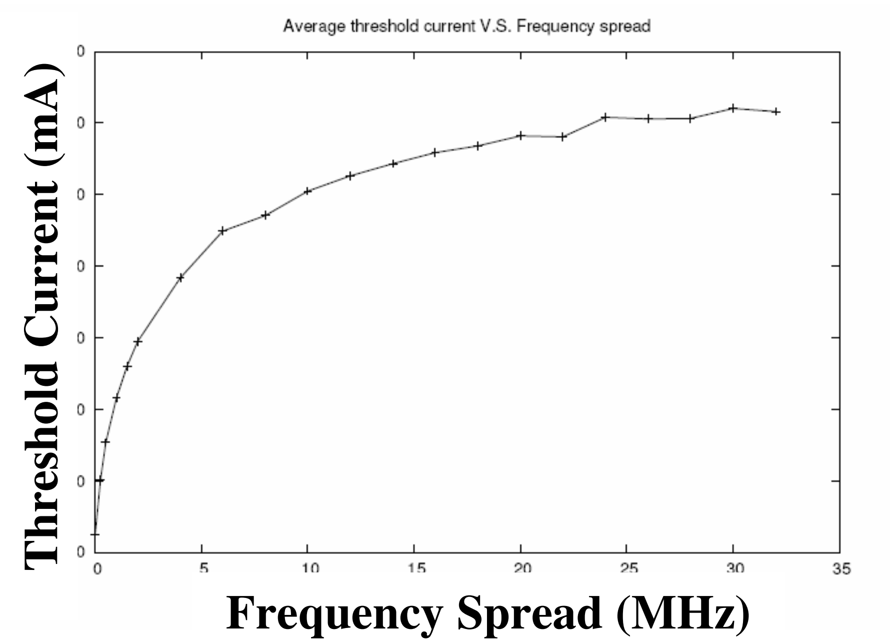
σ_f[MHz]	I_{th}[mA]		σ_I[mA]	
	mode 1	mode 1-4	mode 1	mode 1-4
0	25.8	25.8	0	0
10.0	427.7	405.5	71.1	68.2

Threshold Current for Polarized Dipole HOMs

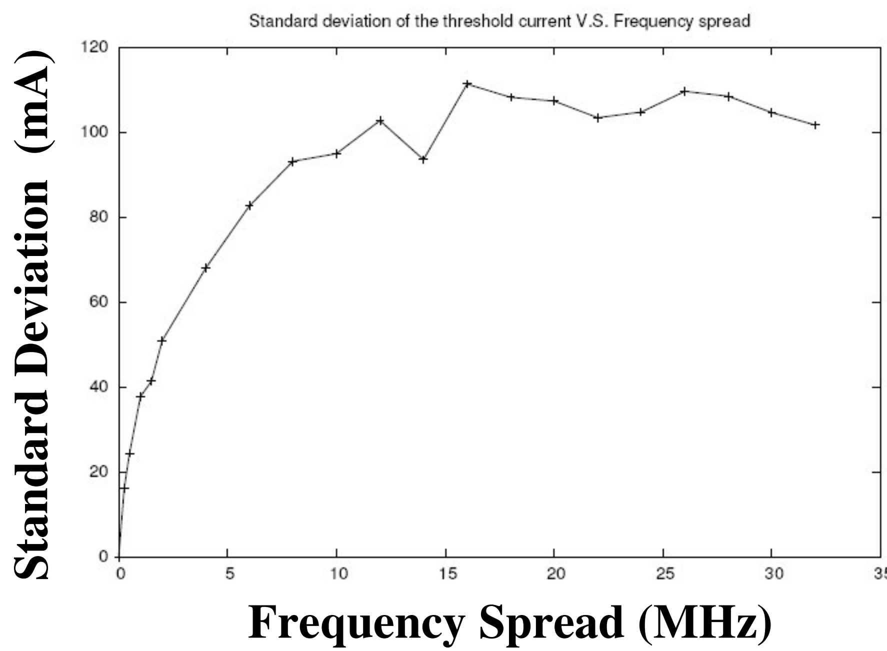
Δf[MHz]	Coupling	σ_f[MHz]	I_{th}[mA]	σ_I[mA]
10	NO	0	25.8	N/A
10	YES	0	93.4	N/A
60	NO	0	25.8	N/A
60	YES	0	117.6	N/A
60	NO	10	409	69
60	YES	10	2227	380

Dipole BBU Threshold Current

Achieving Higher Threshold Current through HOM Frequency Randomization



Introducing a random distribution of HOM frequencies can increase the threshold current.



The width of the threshold current distribution forms a smooth curve.

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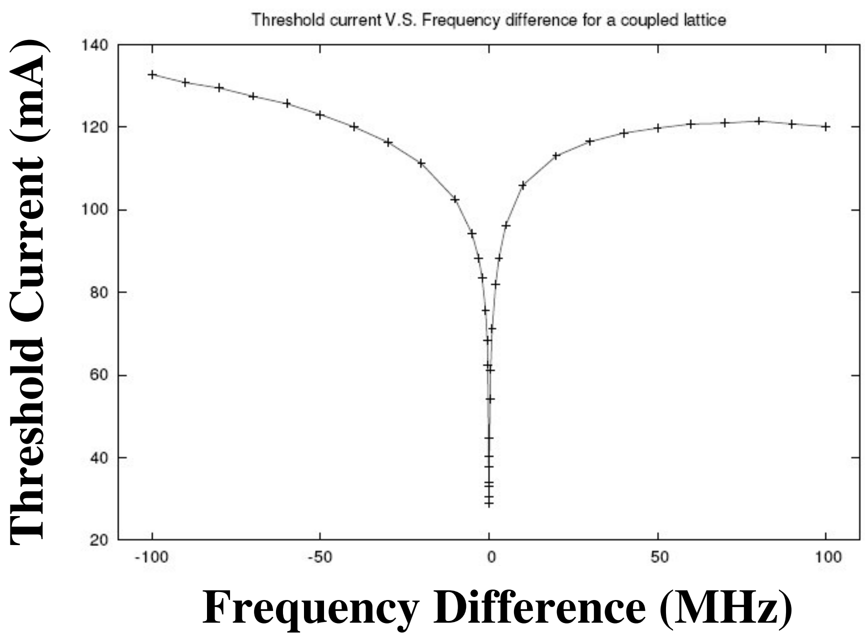
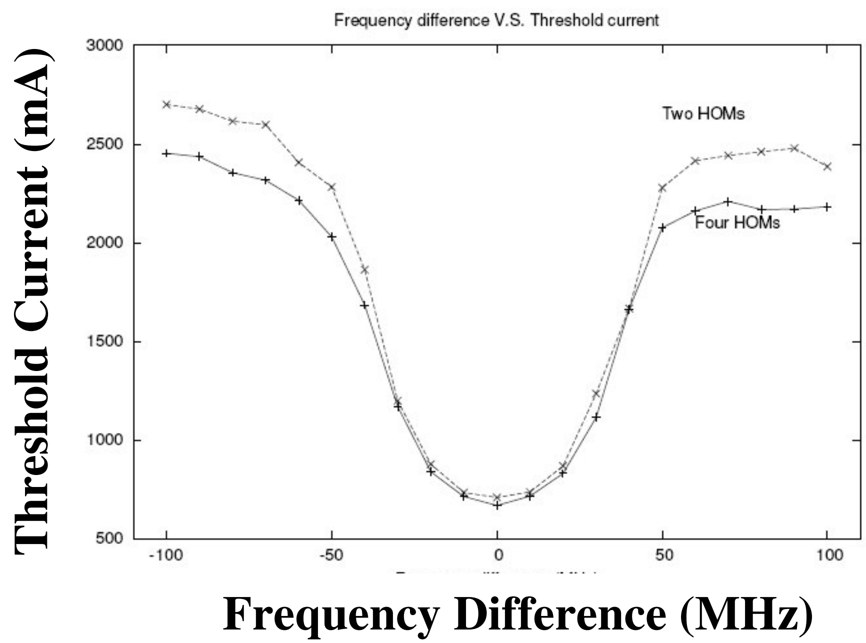
Dipole BBU Threshold

The Effect of Polarizing HOMs and Randomizing HOM Frequencies
50 MHz frequency separation – Threshold current increases by 4 times.
10 MHz frequency spread -- Threshold current increases by 16 times.



Without Frequency Spread



With 10MHz Frequency Spread

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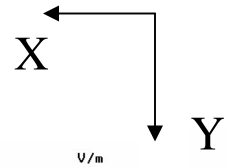
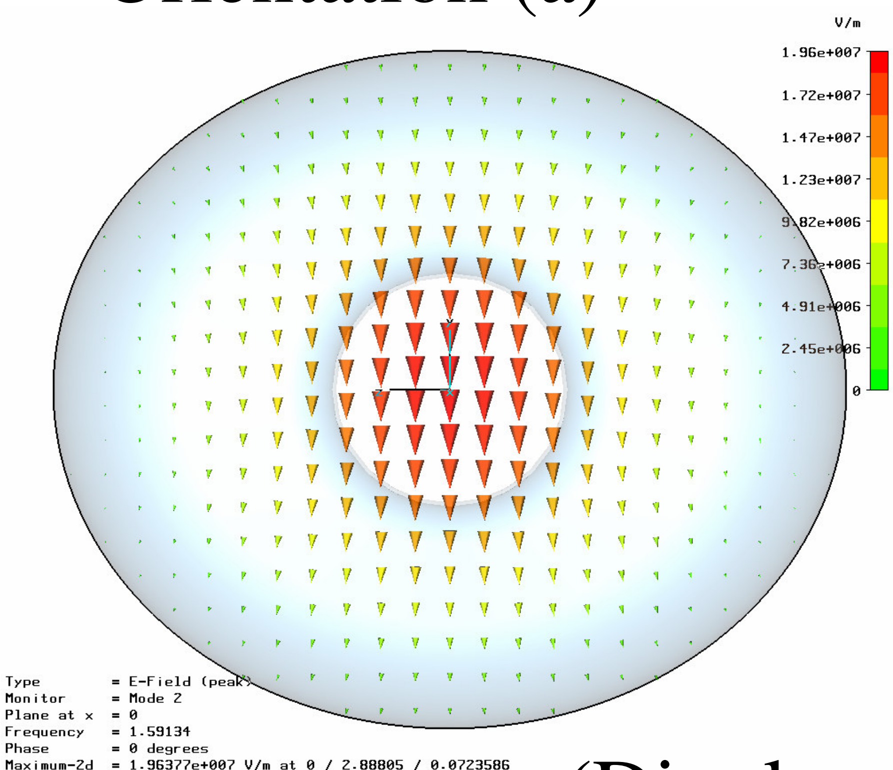
RF Cavity with Polarized Dipole HOMs

Different polarized orientation

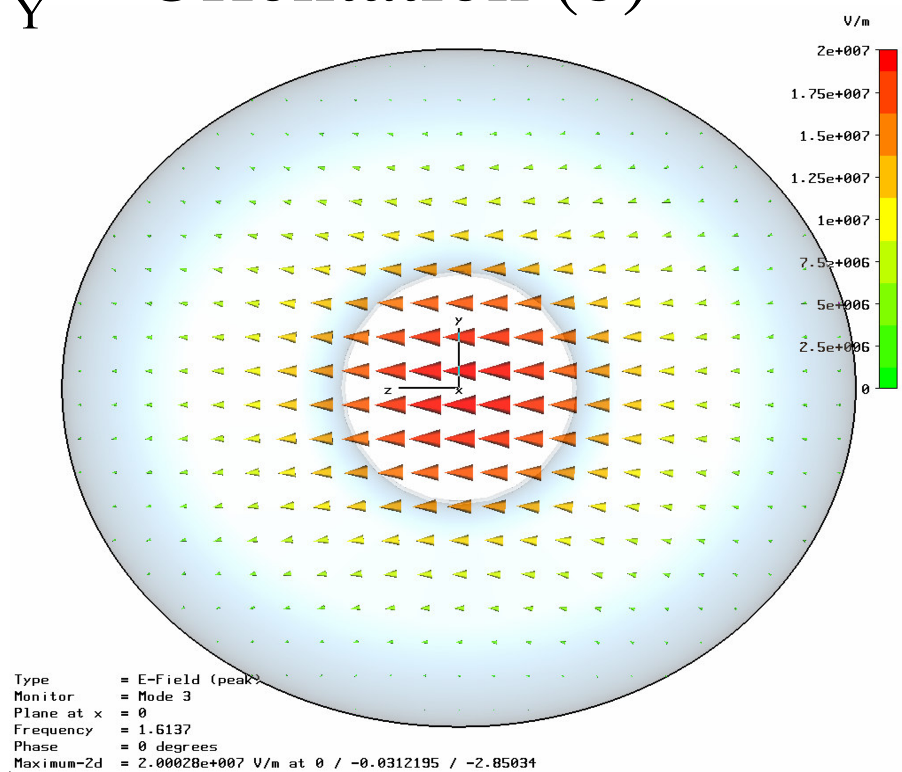
R(horizontal) : R(vertical) = 19 : 12



Orientation (a)



Orientation (b)



(Dipole mode TE111)

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Quadrupole BBU Instability

Approximate Formula of the Threshold Current For a Single Quadrupole HOM

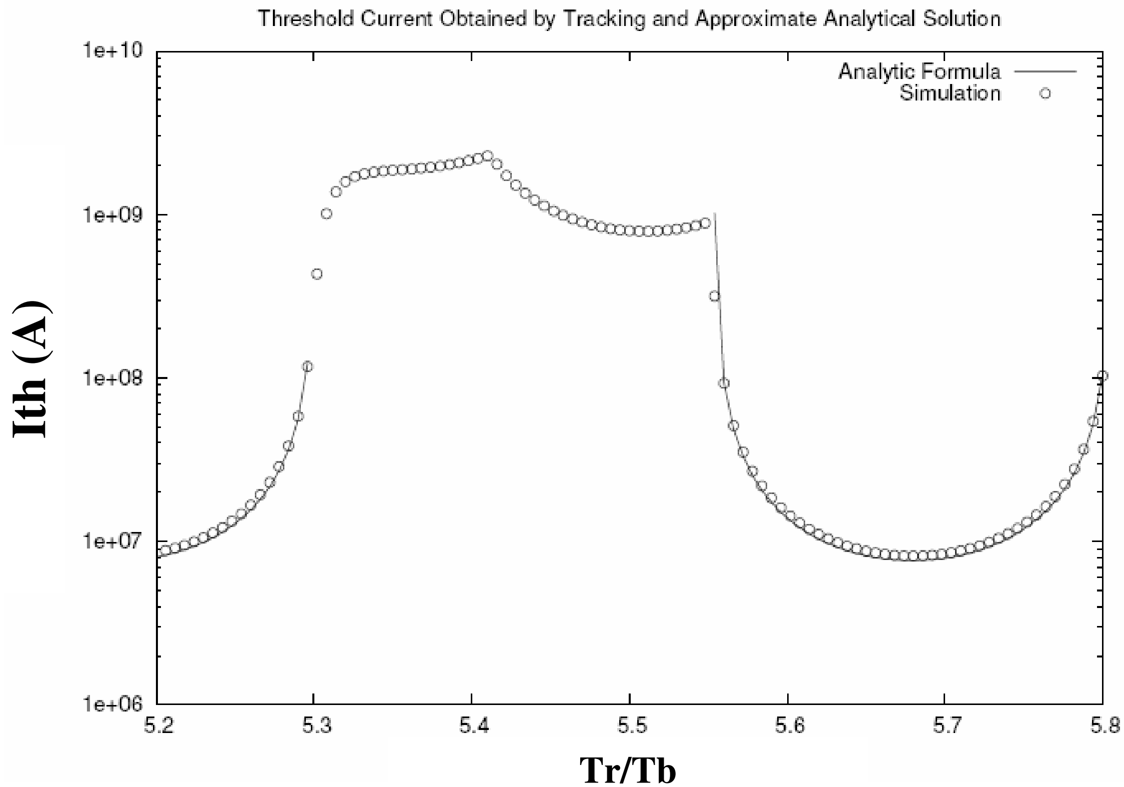
$$I_0 = -\frac{\omega_\lambda \gamma E_e}{2\frac{ec}{r_0^4}\left(\frac{R}{Q}\right)_\lambda Q_\lambda \varepsilon_n} \frac{1}{\beta_{x1}\beta_{x2}\sin 2\Delta\psi_x + \beta_{y1}\beta_{y2}\sin 2\Delta\psi_y} \frac{1}{\sin\omega_\lambda t_r}$$

Simulation vs. Analytic Formula

$$\omega = 2.575\text{GHz}$$

$$(R/Q) = 34\Omega$$

$$Q = 1.0\times10^4$$



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Quadrupole BBU for ERL

Dominant Quadrupole HOMs in the 7-cell ERL Cavity

	f_λ[GHz]	Q_λ/Q_0	$(R/Q)_\lambda[\Omega/\text{cm}^2]$	$(R/Q)_\lambda[\Omega]$	$(R/Q)_\lambda \cdot Q_\lambda[\Omega]$
1	2.3052	0.570	0.052267	0.96196	5.4832×10^9
2	2.3074	0.572	0.045267	0.82995	4.7473×10^9
3	2.4896	0.516	0.060044	0.81231	4.1915×10^9
4	3.2414	0.256	0.154078	0.72540	1.8570×10^9
5	3.2532	0.259	0.344944	1.60056	4.1454×10^9
6	3.2670	0.263	0.217078	0.99034	2.6046×10^9
7	3.4860	0.315	0.106633	0.37527	1.1821×10^9
8	3.5144	0.328	0.049389	0.16826	5.5190×10^8
9	3.8531	0.251	0.061756	0.14561	3.6549×10^8

Simulation Results for the Four Most Important HOMs (1,2,3 and 5)

	$Q_\lambda \cdot 10^{-3}$	Q_λ
Single Cavity at Low Energy	50A	50 mA
Full ERL Lattice without Frequency Spread	200A	200 mA
Full ERL Lattice with 10 MHz Frequency Spread	1200A	1.2A

Conclusions

1. Randomizing the HOM frequency is by far the most effective method of increasing the threshold current.

2. Polarizing HOMs can increase the threshold current in the ERL by about 4 times, but strong cavity deformations are required.

3. The threshold current from these three types of BBU is sufficient for our ERL design.