Undulator Radiation Calculations

- Interpretation of SRW results
- Independent Lienerd-Wiechert code
- Comparison with literature

Running SRW

- Fairly easy to build working code from provided examples
- For propagating through lenses, need 2D array – want field at center, but need to be careful, since dimensions of array change in propagation

SRW Interpretation

- Naturally gives results in units of photons/sec/mm^2/0.1%BW
- By Matt's advice, use current of 1 e-/sec, so units are photons/electron/mm^2/0.1%BW
- It also gives “fields”, defined so that squaring them gives funny units above – contain phase information

SRW Interpretation

- Can trivially get intensity in units of photons/electron/m^2/BW by multiplying by 10^9
- Note that bandwidth is photon energy, so these units are the same as J/electron/m^2/J

Aid from Jackson

- Jackson 3rd edition, pg 673-674 very useful – in the end, he finds:

$$\frac{d^2 I}{d\omega\, d\Omega} = 2\, |\mathbf{A}(\omega)|^2$$

- A is proportional to electric field, up to factors of R, ε_0, and c
- Derivation assumes certain Fourier conventions – for SRW, the 1/2π factor comes when converting back to real space, but we can rederive Jackson's formula with the new convention
- Use factor of ħ to convert SRW intensity from energy-space to frequency space

Real Electric Field

- $\tilde{E}(\omega) = \mathrm{sqrt}(I_{SRW}\ 10^9\ \hbar/\varepsilon_0/c)$

- $E(t) = 1/\pi \int^{\infty}_{0} \tilde{E}(\omega) \cos(\omega t)\, d\omega$

  (standard Fourier integral, assuming E(t) is real, so ignore negative frequencies and multiply by 2)

- $E(t) = 1/\pi/\hbar \int^{\infty}_{0} \tilde{E}(\omega) \cos(e/\hbar\ t)\, de$

Electric Field Longitudinal and Frequency Dependence

- In SRW, simulate frequencies lying between the lowest frequency we see off-axis, and the highest frequency on-axis – obtain electric fields at the various z positions behind the lenses

- Obtain z dependence with computation of ponderomotive phase between electron and light at each z position – also, subtract phase of SRW light (assuming all light and electrons are in-phase at center of 2^{nd} undulator) – essentially same as using $\cos(kz - \omega t + \Phi)$

Lienerd-Wiechert Code

- Frustrated with SRW, so wrote my own code
- Electron motion in an undulator described by Schmüser et al, “Ultraviolet and Soft X-Ray Free-Electron Lasers”, eqtns A.20 (planar) and A.27 (helical)

Lienerd-Wiechert Code (cont.)

- For each point on the lens, and for each time step in tracking the electron, compute Lienerd-Wiehcert field (Jackson, eqtn 14.14)

$$\mathbf{E}(\mathbf{x}, t) = e\left[\frac{\mathbf{n} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta}\cdot\mathbf{n})^3 R^2}\right]_{\text{ret}} + \frac{e}{c}\left[\frac{\mathbf{n} \times \{(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \boldsymbol{\beta}\cdot\mathbf{n})^3 R}\right]_{\text{ret}}$$

- Also, determine time when said field is seen by the lens – equally-spaced time steps in electron tracking are not equally spaced when observing its field

Propogation Through Lens System

- Get strength of first harmonic by doing integral of its product with pure sine and cosine waves having frequency of 1st harmonic (value of frequency varies if off-axis)

- From Lebedev, OSC paper ICFA Beam Dynamics Newsletter, pg 112:

$$E(r'') = \frac{1}{2\pi i c} \int_S \frac{\omega(\theta) E_\omega(r)}{|r'' - r|} e^{i\omega(\theta)|r'' - r|/c} \, ds$$

Propogation Through Lens System

- If perfect focusing, can ignore phase factors by definition

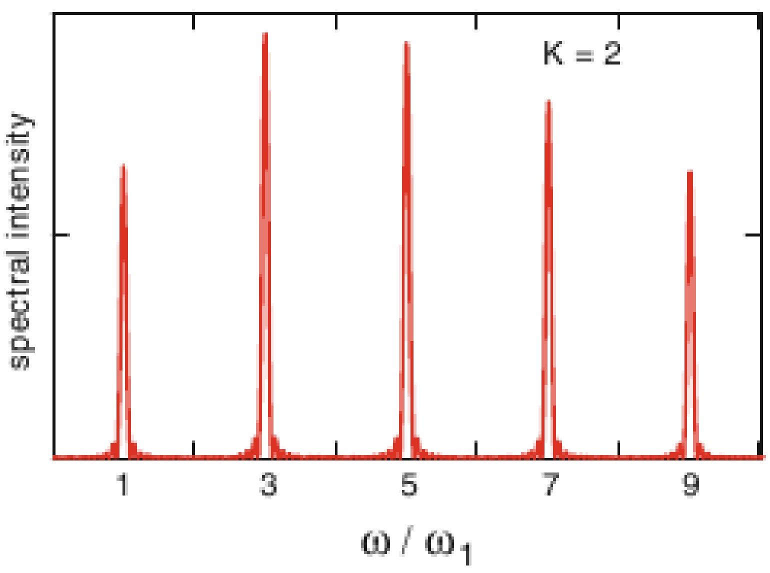
- When getting energy transfer for planar undulator, add in Bessel function factor to account for varying longitudinal velocity (refer back to Lebedev's paper if interested)

- Can force a square lens for SRW comparisons

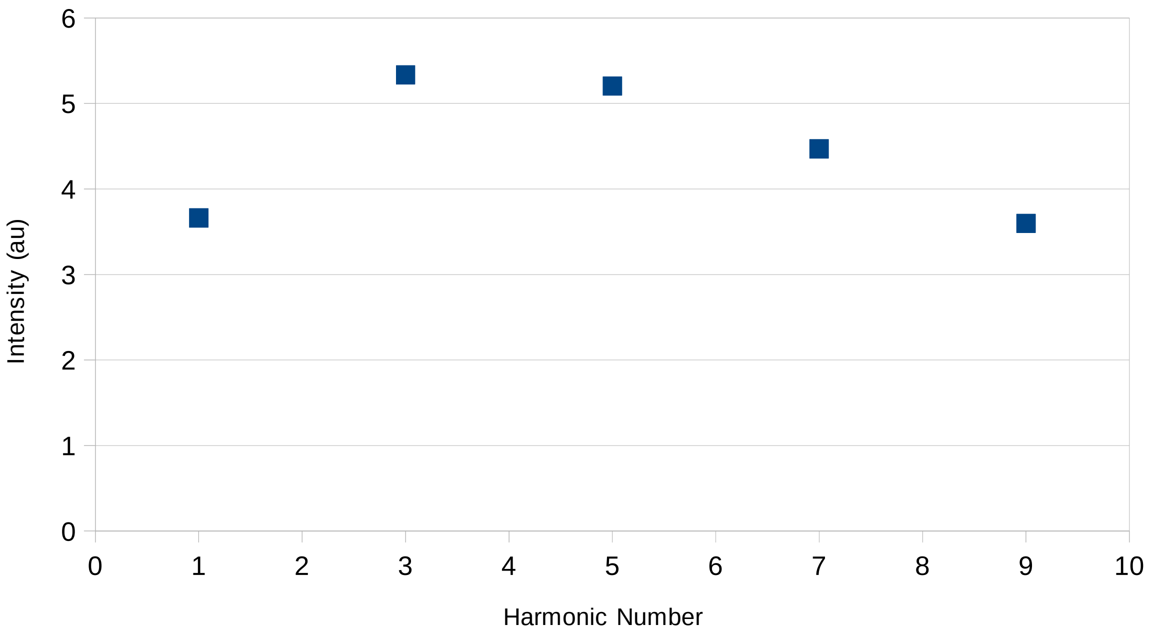
Consistency Checks

- SRW, Lienerd-Wiechert Code, and Lebedev (cited earlier) never had any significant disagreements – see backup slides
- Note that Lebedev's formulas assume a circular lens, and I've only been able to get SRW to use a rectangular lens – also, SRW has extra matching sections at undulator's ends

Planar Undulator Harmonics K=2



Schmüser et al



Lienerd-Wiechert Code

On-Axis Helical Undulator Energy/e- vs K Value

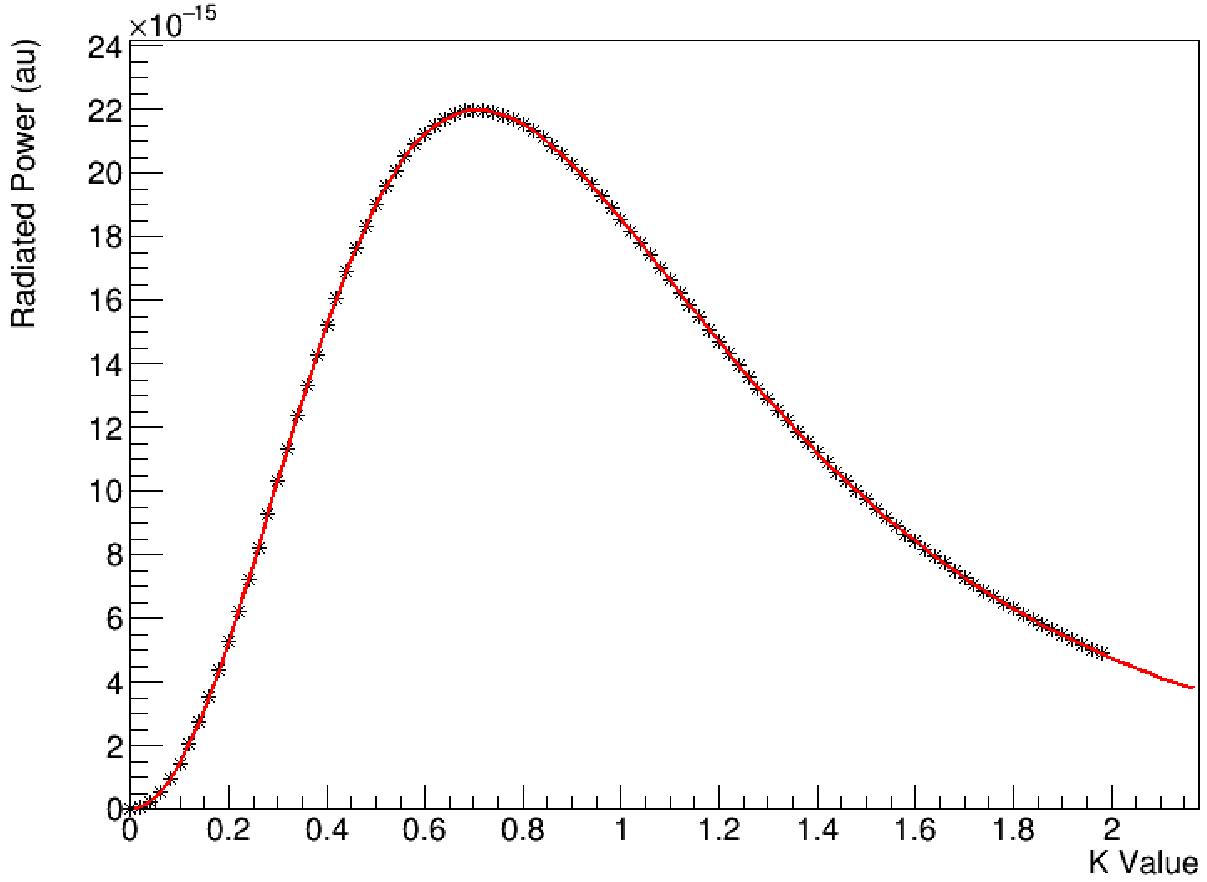
- Kincaid (Jour. App Phys, 48, 7, July 1977, p2691) claims on-axis energy radiated per e- scales as $K^2/(1+K^2)^3$ (note eqtn 24 of main text leaves out K^2 in numerator)

$$\frac{dW}{d\Omega} = \frac{Ne^2\omega_0 K^2}{c} \frac{8\gamma^4}{(1+K^2+\gamma^2\theta^2)^3}$$

$$\times \sum_{n=1}^{\infty} n^2 \left[J_n'^2(x_n) + \left(\frac{\gamma\theta}{K} - \frac{n}{x_n} \right)^2 J_n^2(x_n) \right].$$

x_n is proportional to θ, so on-axis, only n=1 term contributes to the sum and is just a constant 1/2

On-Axis Helical Undulator Energy/e- vs K Value



Plot is for Lienerd-Wiechert code (SRW gives similar results)
Fit to form $A*K^2/(1+K^2)^3$ shows no visible deviation

Conclusion

SRW and Lienerd-Wiechert code give results which compare well with:

- One another
- Lebedev's analytic calculations
- A standard FEL text
- Kincaid's original paper

Backup Slides

500 MeV Results

	Peak Field (V/m)	Energy Transfer (meV)
SRW – telescope Square lens, 16mm/side	38	93
SRW – lenses as above, Ignore extra bit of undulator	38	81
Lebedev - circular lens, radius 8mm	35	79
Lebedev - circular lens, radius 8 x sqrt(2) mm	41	93
L-W code – square lens, 16mm/side	38	85
L-W code – circular lens, 8mm radius	35	80

1 GeV Results (Planar Undulator)

	Peak Field (V/m)	Energy Transfer (meV)
SRW – telescope Square lens, 16mm/side	38	95
SRW – lenses as above, Ignore extra bit of undulator	38	83
Lebedev - circular lens, radius 8mm	35	81
Lebedev - circular lens, radius 8 x sqrt(2) mm	42	96
L-W code – square lens, 16mm/side	38	

1 GeV, Helical Undulator

	Peak Field (V/m)	Energy Transfer (meV)
SRW – telescope Square lens, 16mm/side	35	164
SRW – lenses as above, Ignore extra bit of undulator	35	148
L-W code – square lens, 16mm/side	35	162

Helical Undulator

	Peak Field (V/m)	Energy Transfer (meV)
4 0.45 m periods K = 3.55	43 (SRW) 46 (LW)	156 (SRW) 152 (LW)
6 0.3 m periods K = 4.41	50 (SRW) 52 (LW)	212 (SRW) 212 (LW)
8 0.225 m periods K = 5.12	54 (SRW) 56 (LW)	254 (SRW) 266 (LW)

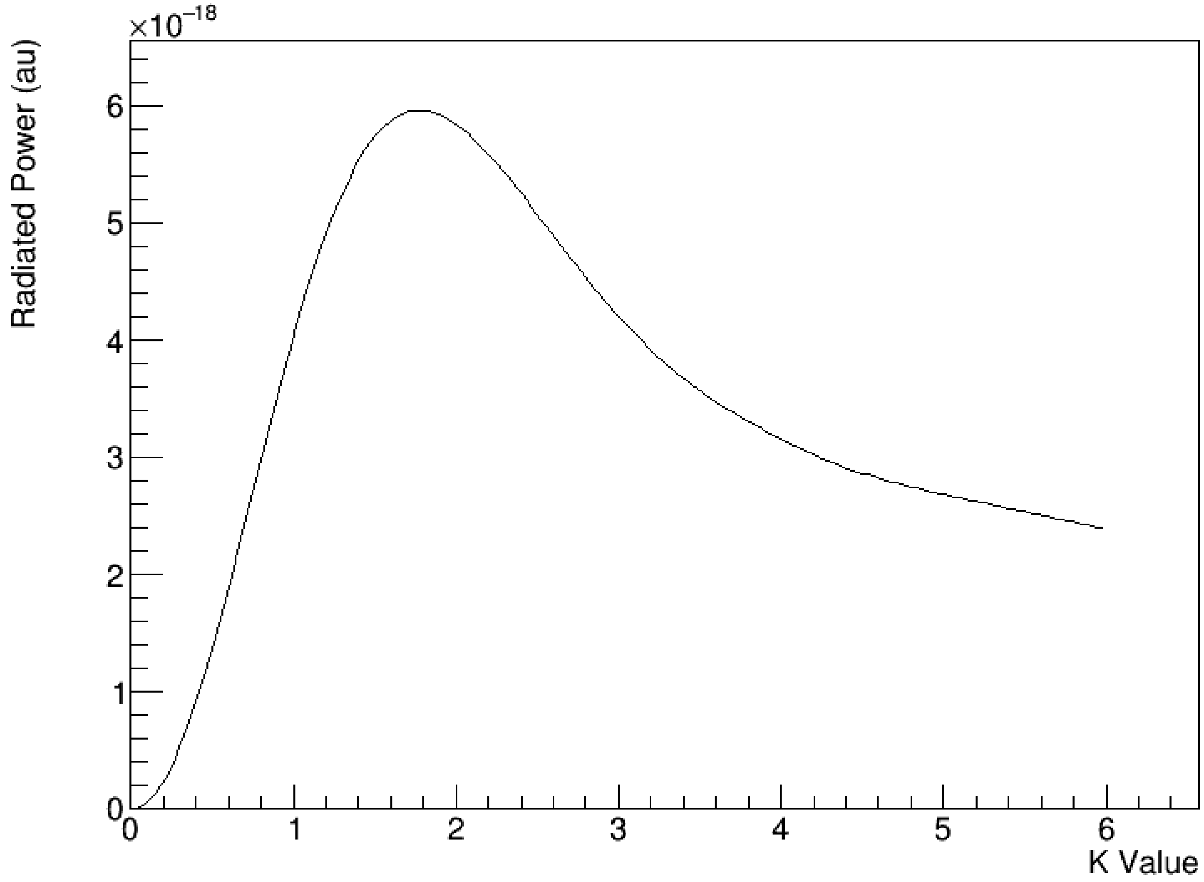
(telescope, square lens, 16mm/side,
1 GeV, 800 nm wavelength)

Planar Undulator

	Peak Field (V/m)	Energy Transfer (meV)
6 0.3 m periods K = 6.23	57 (SRW) 56 (SRW)	126 (SRW) 114 (LW)
8 0.225 m periods K = 7.24	63 (SRW) 60 (LW)	150 (SRW) 141 (LW)

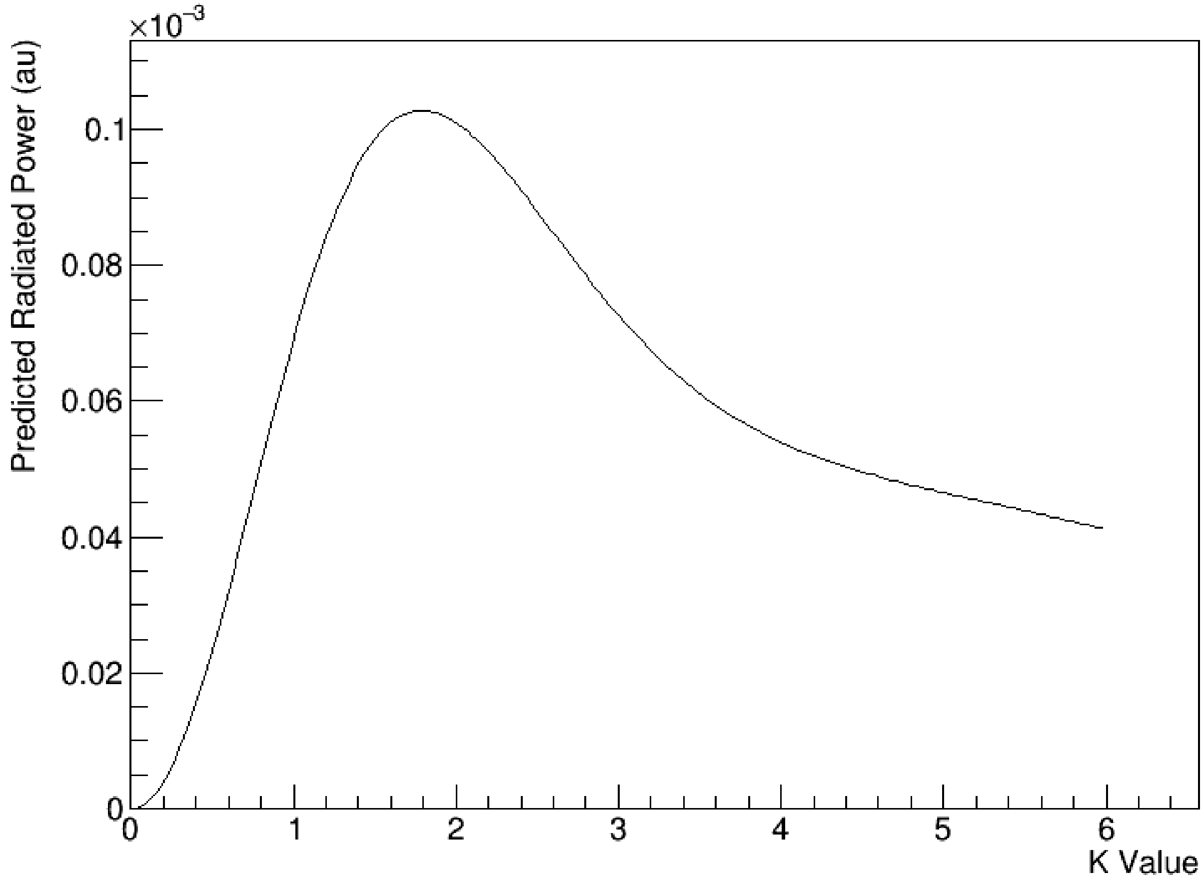
(telescope, square lens, 16mm/side,
1 GeV, 800 nm wavelength)

Off-Axis Helical Undulator Energy/e- vs K Value



Plot is for Lienerd-Wiechert code

Off-Axis Helical Undulator Energy/e- vs K Value



Plot is for Kincaid's formula