$$
\begin{aligned}
\Delta s_x &\sim (M_{51} x_p + M_{52} x'_p). \\
\Delta \phi_x &\sim k(M_{51} x_p + M_{52} x'_p). \\
\Delta \phi_x &\sim k(M_{51}(x_{p\beta} + x_e) + M_{52}(x'_{p\beta} + x'_e). \\
\Delta \phi_x &\sim k(M_{51}(x_{p\beta} + \eta\delta) + M_{52}(x'_{p\beta} + \eta'\delta). \\
\Delta \phi_x &\sim k(M_{51} x_{p\beta} + M_{52} x'_{p\beta} + (M_{51}\eta + M_{52}\eta')\delta).
\end{aligned}
$$

$$\text{set } (M_{51}\eta_p + M_{52}\eta'_p)\delta) = 0.$$

Then

$$\Delta \phi_x \sim k(M_{51} x_{p\beta} + M_{52} x'_{p\beta}).$$

$$
\begin{aligned}
x_{p\beta} &= a\sqrt{\beta_p}\cos\theta(s) \\
x'_{p\beta} &= \frac{1}{2}\frac{a\beta'_p}{\sqrt{\beta_p}}\cos\theta(s) - \frac{a}{\sqrt{\beta_p}}\sin\theta(s) \\
&= -\frac{a}{\sqrt{\beta_p}}(\alpha_p\cos\theta(s) + \sin\theta(s))
\end{aligned}
$$

$$
\Delta s = a\left(M_{51}\sqrt{\beta}\cos\theta - M_{52}\frac{(\alpha\cos\theta + \sin\theta)}{\sqrt{\beta}}\right)
$$

The momentum change in the kicker is

$$
\Delta p/p = \xi\sin(k\Delta s).
$$

At the kicker $\Delta x_{k\beta} = -\eta_k \Delta p/p$ and $\Delta x'_{k\beta} = -\eta'_k \Delta p/p$.

$$a = \beta {x'}^2 + \gamma x^2 + 2\alpha x x'$$

$$\Delta a = -2\Delta p/p(\beta x'_{k\beta}\eta'_k + \gamma x_{k\beta}\eta_k + \alpha(x_{k\beta}\eta'_k + x'_{k\beta}\eta_k))$$

If phase advance is 180 deg. then

$$2(\Delta p/p)a\left(\eta'_k(-\sqrt{\beta}\sin\theta) + \eta_k(\frac{\cos\theta - \alpha\sin\theta}{\sqrt{\beta}})\right)$$

$$\Delta a = 2a\xi \sin(k\Delta s)\left(\eta_k'(-\sqrt{\beta_k}\sin\theta) + \eta_k(\frac{\cos\theta - \alpha_k\sin\theta}{\sqrt{\beta_k}})\right)$$

If $k\Delta s \ll \pi/2,$

And average over betatron phases

$$\langle\Delta a\rangle = -a^2\xi k\left(M_{51}\eta_k\sqrt{\frac{\beta_p}{\beta_k}} + M_{52}\left(\frac{\eta_k(\alpha_p - \alpha_k)}{\sqrt{\beta_p\beta_k}} + \eta_k'\sqrt{\frac{\beta_k}{\beta_p}}\right)\right)$$

If optics are symmetric

$$\langle\Delta a\rangle = -a^2\xi k\left(M_{51}\eta_k + M_{52}\left(2\frac{\eta_k(\alpha_k)}{\beta_p} + \eta_k'\right)\right)$$