Alexander M, Feb. 9 2018

ESTIMATION OF IBS AT CESR AT 500 AND 300 MeV

Emittance growth defined by [see CBN 03-11 for details]

$$\frac{d\varepsilon_{x,y}}{dt} \cong \left\langle \left(H_{x,y} + \frac{\beta_{x,y}}{\gamma^2} \right) \frac{d(\Delta E/E)^2_{tot}}{dt} \right\rangle - 2\alpha_{x,y}\varepsilon_{x,y},$$

where dispersion invariant defined as

$$H_{x,y} = \frac{1}{\beta_{x,y}} \left(\eta^2_{x,y} + (\beta_{x,y}\eta'_{x,y} - \frac{1}{2}\beta'_{x,y}\eta_{x,y})^2 \right),$$

$\eta_{x,y}$ –are dispersion functions. Partial decrements $\alpha_{x,y,s}$ defined as $\alpha_i = \frac{J_i}{2\tau_s}$,

where $J_x \cong 1, J_y = 1, J_s \cong 2,\ J_x + J_s = 3$.

Partial decrement for energy spread is the same as for the emittance.

The rate of energy spread growth includes additive component from IBS .

$$\frac{d(\Delta E/E)^2_{tot}}{dt} = \frac{d(\Delta E/E)^2_{IBS}}{dt} + \frac{d(\Delta E/E)^2_{\gamma}}{dt} - \alpha_s \left(\frac{\Delta E}{E} \right)^2,$$

INTRA-BEAM SCATTERING

Collisions inside moving bunch equalize temperature; the same processes responsible for the shortening of a beam lifetime.

The temperature of electron gas can be expressed as the following

$$\tfrac{3}{2} N k_B T \cong N \cdot mc^2 \gamma \left[\frac{\gamma \varepsilon_x}{\beta_x} + \frac{\gamma \varepsilon_y}{\beta_y} + \gamma \left(\frac{1}{\gamma^2} - \langle \psi \rangle \right) \left(\frac{\Delta p_{\parallel}}{p_0} \right)^2 \right], \qquad \psi = \frac{\gamma}{l} \frac{\partial l}{\partial \gamma}$$

Longitudinal part of temperature has this form because the longitudinal mass is

$$\frac{1}{m_{\parallel}} = \frac{1}{m\gamma} \left(\frac{1}{\gamma^2} - \alpha \right), \ \alpha = \left\langle \frac{\gamma}{l} \frac{\partial l}{\partial \gamma} \right\rangle, \ \alpha = \langle \psi \rangle$$

For values as

$$\beta_{x,y} \cong 10m \,,\ l_b \cong 1cm \,,\ \Delta p / p \cong 5 \cdot 10^{-4} \,,\ \gamma \varepsilon_s \cong 310^{-4}\, cm \cdot rad \,,\ \gamma \varepsilon_y \cong 3 \cdot 10^{-6}\, cm \cdot rad$$

$$\left| \tfrac{3}{2} k_B T \right| \cong mc^2 \gamma \left| 3 \cdot 10^{-7} + 3 \cdot 10^{-9} - 4 \cdot 10^{-11} \right|.$$

One can see that the longitudinal temperature is the lowest one and it is negative above the critical energy (for CESR $\alpha \approx 0.011 \rightarrow \gamma_{cr} \approx 10$).

Other important moment is that during equalizing the vertical emittance becomes rising even without coupling associated with imperfections of magnetic structure.

In a moving frame, the velocity of transverse motion is dominant and the speed of diffusion can be expressed by simple formula

$$\frac{dp'^2}{dt'} = \frac{4\pi e^4 n' Ln_C}{v'},$$

where $Ln_C = ln\frac{a_{max}}{a_{min}} \cong ln\sqrt{\frac{(v'/c)^6}{4\pi r_0^3 n'}}$ is Coulomb's integral, n' is the density in the moving frame, v' stands for speed of transverse motion in moving frame. Transforming in the Lab frame

$$\frac{d(\Delta p_{\parallel}/p)^2}{dt} \cong \frac{d(\Delta E/E)^2}{dt} = \sqrt{\frac{2}{\pi}}\frac{Ln_C N r_0^2 c}{\gamma^3 \varepsilon_x \cdot \sqrt{\varepsilon_y \beta_y}\sigma_s \cdot \sqrt{1+\frac{(\eta \Delta p_{\parallel}/p)^2}{\varepsilon_x \beta_x}}}.$$

For simplest FODO structure solution of this equation can be expressed as

$$\varepsilon_x \cong l \cdot \left(\frac{N r_0^2 c \tau_x L n_C}{4 \kappa_0 \gamma^3 \sigma_s R^2} \right)^{0.4} \quad , \qquad (2l\text{- is period of FODO})$$

where coupling $\kappa_0 = \sqrt{\varepsilon_y / \varepsilon_x}$ defined the the square root of emittance ratio.
In some publications under this name now in use the square of this value.

So the IBS generates coupling what is

$$\kappa_{IBS}^2 \cong \frac{\langle \beta_y \rangle}{\gamma^2 \left\langle H_x \cdot \sqrt{\frac{1}{\beta_y}} \right\rangle} \; .$$

For FODO structure this can be estimated as $\kappa_{IBS} \cong \frac{R}{\gamma l}$. Geometrical coupling defined by rotation of quads by random angle within amplitude ϑ_0. So resulting coupling coefficient comes to

$$\kappa^2 = \kappa_0^2 + \kappa_{IBS}^2 \, .$$

For vertical emittance

$$\varepsilon_y \cong \left(\frac{N r_0^2 c \tau_x L n_C}{4\gamma^3 \sigma_s R^2} \right)^{0.4} \gamma \cdot l \cdot \left(\kappa_0^2 + \kappa_{IBS}^2 \right)^{0.8} =$$

$$= \left(\frac{N r_0^2 c \tau_x L n_C}{4\gamma^3 \sigma_s R^2} \right)^{0.4} \gamma \cdot l \cdot \left(\vartheta_0^2 \, 2\pi R l^{1/4} + \frac{R^2}{\gamma l^{3/4}} \right)^{0.8} .$$

After reminding these formulas, on the next page there are results of numerica calculations. CESR approximated just by regular structure.

Numerical calculation carried with beta functions corresponding the regular structure.

options for sharin

	A	B	C	D	S	T	U
1	0,1	2,130E+00	8,880E-05	2,25E-08	1,700	3,37E-05	3,11E-09
2	0,5	2,820E+00	1,545E-04	3,92E-08	2,240	5,87E-05	5,41E-09
3	1	3,170E+00	1,960E-04	4,98E-08	2,520	7,45E-05	6,86E-09
4	2	3,580E+00	2,488E-04	6,31E-08	2,840	9,46E-05	8,71E-09
5	3	3,840E+00	2,859E-04	7,26E-08	3,050	1,09E-04	1,00E-08
6	4	4,030E+00	3,157E-04	8,01E-08	3,207	1,20E-04	1,11E-08
7	5	4,180E+00	3,408E-04	8,65E-08	3,330	1,30E-04	1,19E-08
8	6	4,320E+00	3,628E-04	9,21E-08	3,440	1,38E-04	1,27E-08
9	7	4,440E+00	3,826E-04	9,71E-08	3,530	1,46E-04	1,34E-08
10	8	4,540E+00	4,005E-04	1,02E-07	3,610	1,52E-04	1,40E-08
11	9	4,630E+00	4,171E-04	1,06E-07	3,690	1,59E-04	1,46E-08
12	10	4,710E+00	4,324E-04	1,10E-07	3,750	1,65E-04	1,51E-08

Copling at zero current is 0.001

Current in mA; RF voltage is 1200kV