An Analytic Model for the e-Cloud in a Magnetic Field Dominated Ring

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Outline

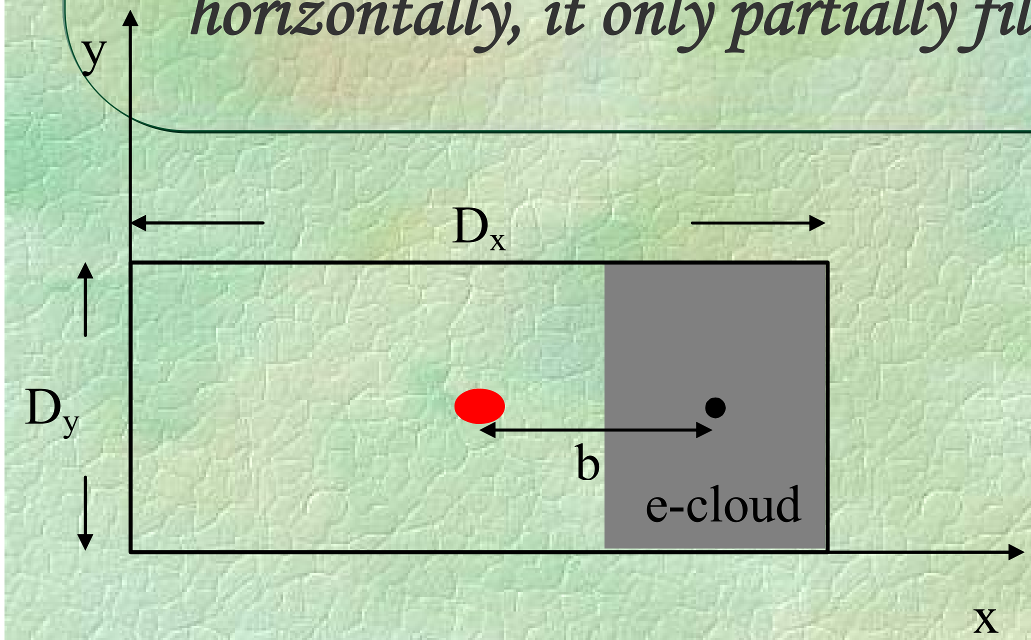
- ✓ *Model Description*
- ✓ *Vertical tune-shift*
- ✓ *Build-up*
- ✓ *Life-time*
- ✓ *Theory and experiment*
- ✓ *Summary and conclusions*

Model Description

- *Along the ring each bunch experiences ,in **average**, a certain cloud density:*

$$q(s) = \sum_{n=-\infty}^{\infty} q_n \exp\left(j2\pi ns / C \right) \simeq q_0$$

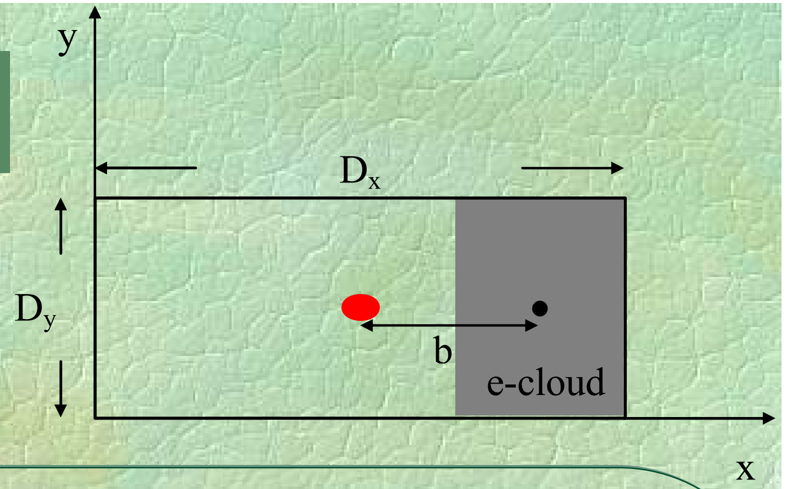
- *This density **varies** from bunch to bunch.*
- ***Effective life-time** is the average life-time of all bunches*
- *The cloud is assumed to have **vertical symmetry** but horizontally, it only partially fills the chamber.*



$$n_{ec} = \frac{q_0}{e} \frac{1}{D_x D_y C}$$

Circumference

Vertical Force of the E-cloud



- *Focusing for positrons (p=+e);defocusing for electrons (p= -e).*

$$\left[\frac{d^2}{ds^2}+\frac{1}{\beta_y^2}\right]\delta y=\frac{pE_y}{mc^2\gamma}=\frac{\delta y}{\beta_{\mathrm{ec}}^2}$$

$$\frac{1}{\beta_{\mathrm{ec}}^2}=-\frac{p}{m_0c^2\gamma}\frac{Q_0}{4\pi\varepsilon_0 C}\frac{f_E\left(\bar{b}=2b/D_x\right)}{D_y^2}$$

- *Uniform cloud. Its center-of-mass at a distance b from center*

$$f_E\left(\bar{b}\right)=\frac{(32/\pi)}{1-\bar{b}}\sum_{n_x,n_y=1}^{\infty}\frac{n_y}{n_x}\frac{\cos\left(\pi n_x\bar{b}\right)-\cos\left(\pi n_x\right)}{n_x^2\left(D_y/D_x\right)+n_x^2\left(D_y/D_x\right)}\sin\left(\frac{\pi}{2}n_x\right)\sin^3\left(\frac{\pi}{2}n_y\right)$$

Relative vertical tune - shift

$$Q_y = \frac{1}{2\pi}\int_0^s ds'\sqrt{\frac{1}{\beta_y^2} - \frac{1}{\beta_{ec}^2}} \simeq \frac{1}{2\pi}\int_0^s ds'\frac{1}{\beta_y} - \frac{1}{2\pi}\frac{1}{2}\int_0^s ds'\frac{\beta_y}{\beta_{ec}^2}$$

$$\delta Q[\%] = 100 \times \frac{Q_y - Q_y^{(0)}}{Q_y^{(0)}} \simeq -\frac{100}{2}\frac{\oint ds \frac{\beta_y(s)}{\beta_{ec}^2(s)}}{\oint ds \frac{1}{\beta_y(s)}} \simeq -\frac{50}{\beta_{ec}^2}\frac{\oint ds\beta_y(s)}{\oint ds \frac{1}{\beta_y(s)}}$$

$$\delta Q[\%] \simeq -50\frac{\beta_{coupling}^2}{\beta_{ec}^2} \simeq \left\{ 50\frac{\beta_{coupling}^2 r_e}{\gamma}\frac{D_x}{D_y}\underbrace{f_E\left(\bar{b}\right)}_{EC\ geometry}\right\} n_{ec}$$

✓ *The relative tune shift is proportional to the **average cloud density**, cloud geometry and the lattice coupling parameter*

$$\beta_{coupling}^2 \equiv \frac{\oint ds\beta_y(s)}{\oint ds \frac{1}{\beta_y(s)}} \simeq \begin{cases} 240.8\left[\mathrm{m}^2\right] & 12\ \text{wigglers} \\ 252.4\ [\mathrm{m}^2] & 6\ \text{wigglers} \end{cases}$$

Build-up

New-born electrons

- "Charging-up" of the chamber: **equilibrium** between generation and absorption at the wall

$$\frac{dn_{ec}}{dt} = -\frac{n_{ec}}{\tau} + \sum_{\nu=1}^{N} n_{\nu}^{(nb)} \delta\left(t - \nu T\right)$$

T - bunch spacing

ν – bunch index

τ – life-time

- Assumption: "new-borns" follow the initial average current of each one of the bunches

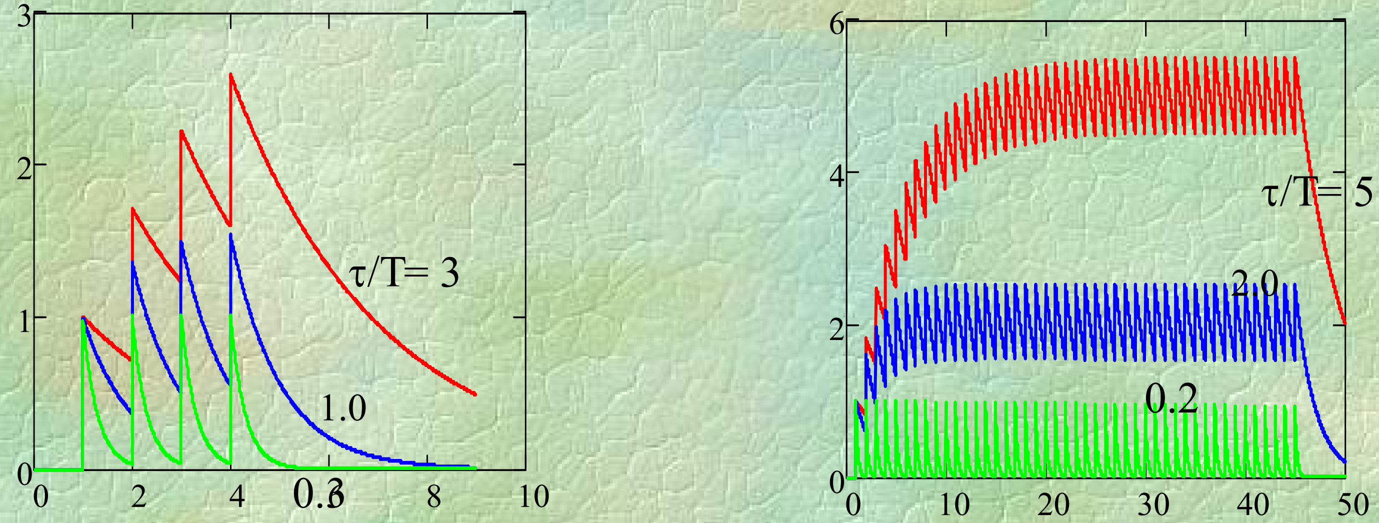
$$\frac{n_{\nu}^{(nb)}}{\left\langle n_{\mu}^{(nb)} \right\rangle_{\mu}} \simeq \frac{I_{\nu}}{\langle I \rangle}$$

- What counts is the **density experienced** by each bunch

$$n_{ec,\mu} \equiv n_{ec}\left(t = \mu T - 0\right) = n_{nb} \sum_{\nu=1}^{N} \frac{I_{\nu}}{\langle I \rangle} h\left(\mu - \nu - 0\right) \exp\left[-\frac{T}{\tau}\left(\mu - \nu\right)\right]$$

Average E-cloud Density

$$\frac{dn_{ec}}{dt} + \frac{n_{ec}}{\tau} = \sum_{\nu=1}^{N} n_{\nu}^{(nb)} \delta\left(t - \nu T\right)$$



✓ *Equilibrium density determined by the* ***life-time*** *!!*

$$n_{ec} \simeq \frac{\tau}{T}\left\langle n_{\nu}^{(nb)} \right\rangle$$

Life-time

$$\tau \cong \frac{D_y}{c}\sqrt{\frac{mc^2}{2E}}$$

- The life-time of a **ballistic** 100[eV] electron is 8nsec (1[eV], 80nsec)
- The dynamics of the electrons **ignoring the kick of the bunches**

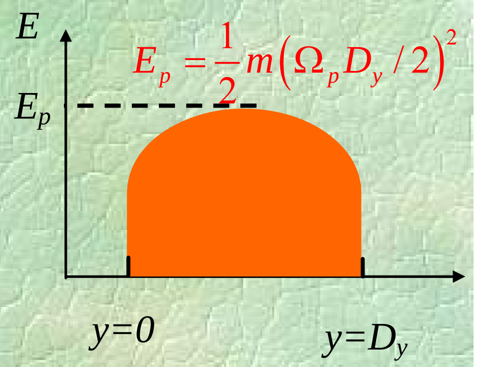
$$\left[\frac{d^2}{dt^2} - \Omega_p^2\right]\delta y_i = 0$$

$$\delta y_i(t) = \sqrt{2E_i / m\Omega_p^2}\,\sinh(\Omega_p t) + (-D_y/2)\cosh(\Omega_p t)$$

$$y_i = D_y/2 + \delta y_i$$

$$\Omega_p^2 = \frac{e^2 n_{ec}}{m\varepsilon_0} f_P\left(\bar{b} = 2b/D_x\right)$$

$$f_P(\bar{b}) = \frac{(2/\pi)^3}{(1-\bar{b})^2}\frac{D_x}{D_y}\sum_{n_x,n_y=1}^{\infty}\frac{n_y}{n_x^2}\frac{\sin^3(\pi n_y/2)\left[\cos(\pi n_x \bar{b}) - \cos(\pi n_x)\right]^2}{n_x^2(D_y/D_x) + n_y^2(D_x/D_y)}$$



Cloud as a potential barrier for the new-borns

Life-time

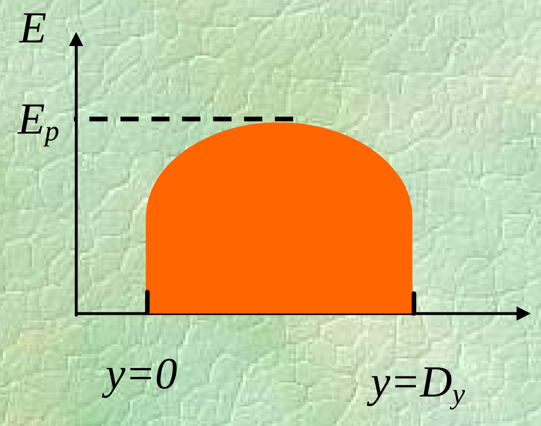
✓ *The dynamics of the electrons* ***ignoring the kick of the bunches***

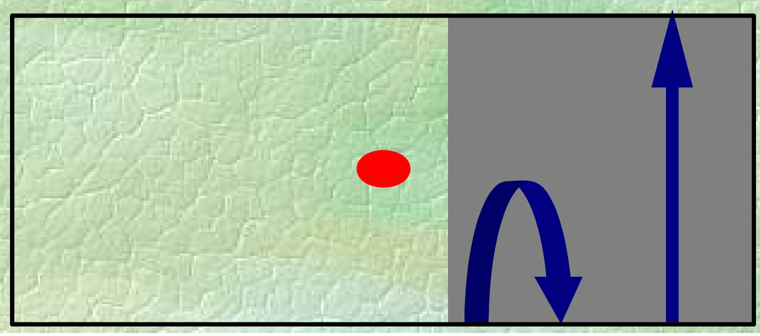
$$D_y/2=\sqrt{2E_i/m\Omega_p^2}\,\sinh\left(\Omega_p\tau\right)+\left(-D_y/2\right)\cosh\left(\Omega_p\tau\right)$$

Potential energy

$$\tau'\left(E_i\right)=\frac{2}{\Omega_p}\begin{cases}\operatorname{atanh}\left(\sqrt{E_p/E_i}\right) & E_i>E_p=\frac{1}{2}m\left(\Omega_p D_y/2\right)^2\\ \operatorname{atanh}\left(\sqrt{E_i/E_p}\right) & E_i<E_p\end{cases}$$

$$\tau=\int_0^{\infty}dE f_{ec}\left(E\right)\tau'\left(E\right)=\frac{1}{E_0}\int_0^{E_0}dE\,\tau'\left(E\right)$$



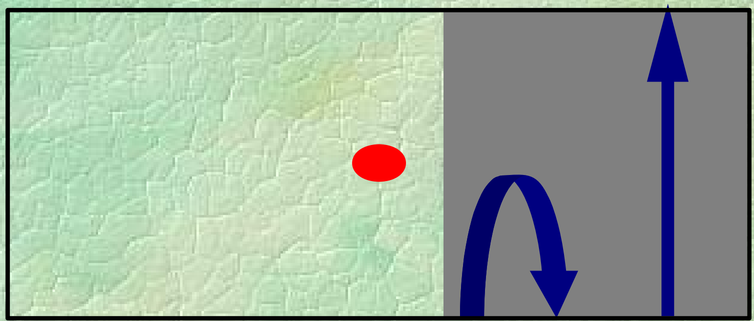


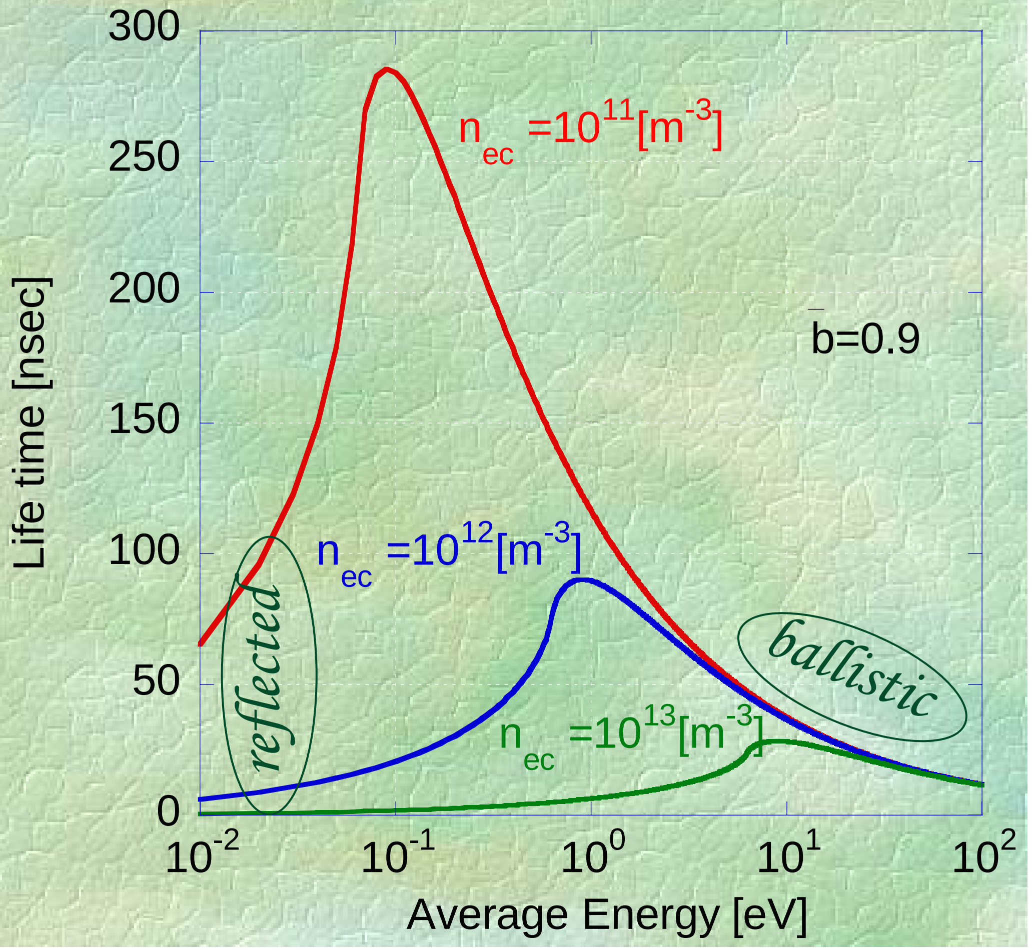
Life-time

✓ *The dynamics of the electrons **ignoring the kick of the bunches***

✓ *Uniform spectrum of electrons* $\langle E \rangle = E_0 / 2$

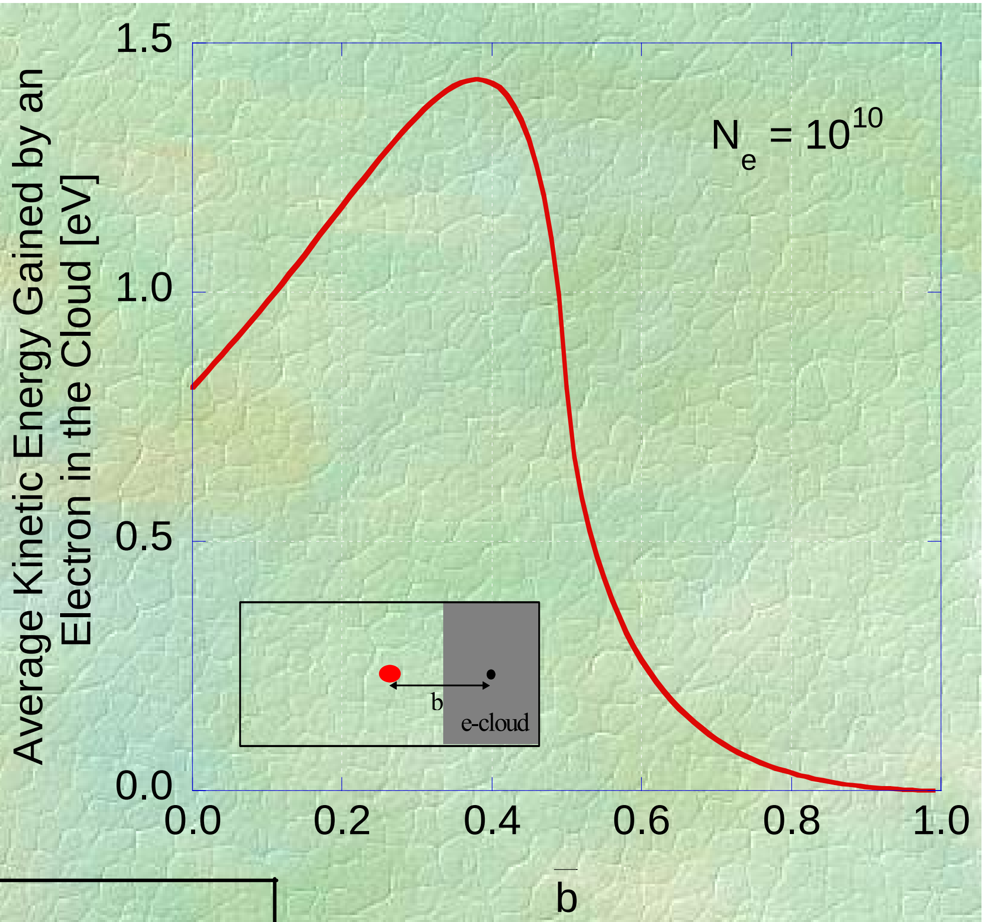
$$\tau = \frac{1}{E_0} \int_0^{E_0} dE \tau'(E)$$





Life-time

- ✓ ***Kick*** *by the positrons bunch.*
- ✓ *Average energy transferred is less than 1.5eV !!* ***No M-P.***
- ✓ *However, this kick may* ***trap the electrons in case of positron bunches.***



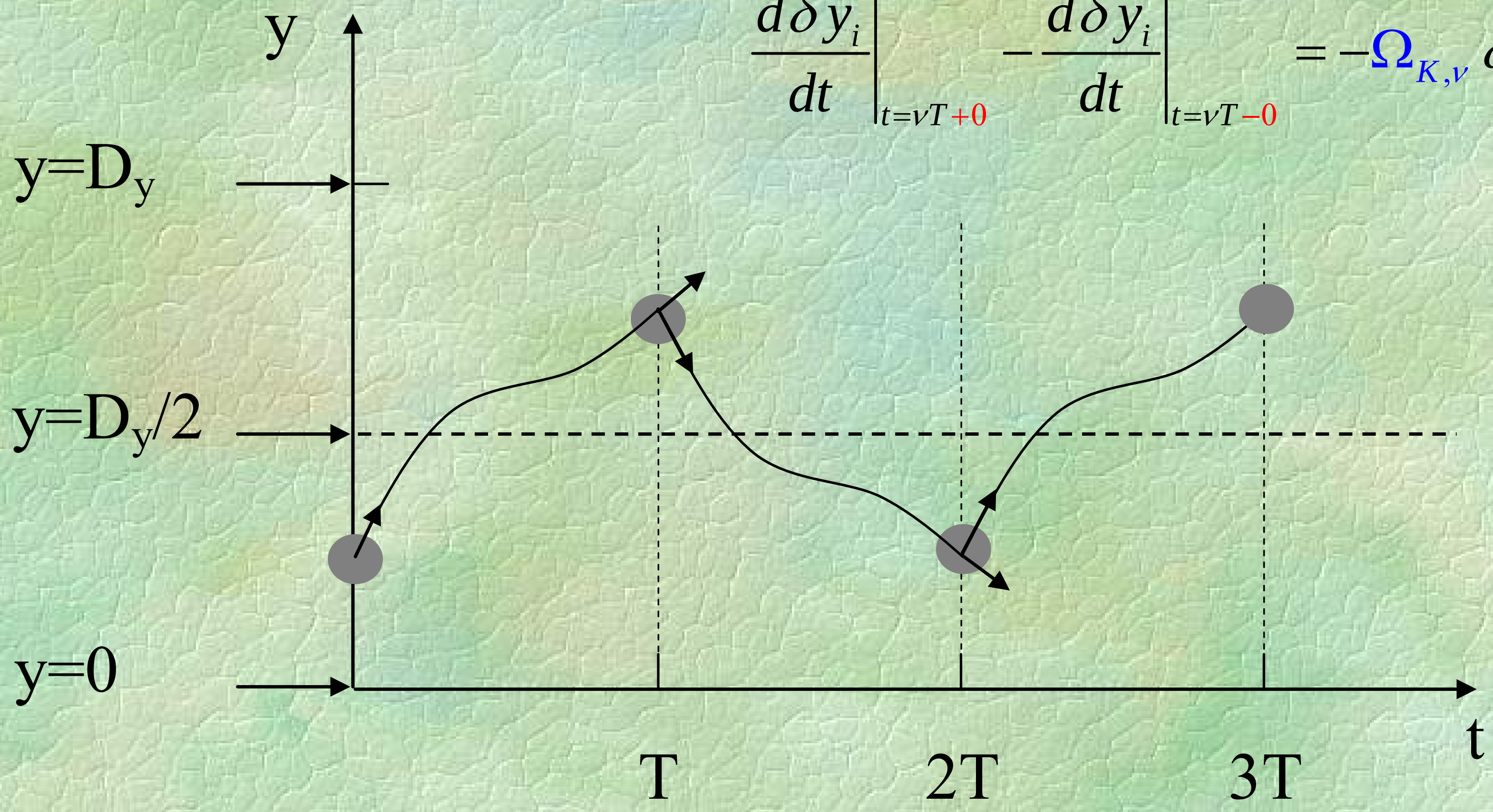
$$\left.\frac{d\delta y_i}{dt}\right|_{t=\nu T+0} - \left.\frac{d\delta y_i}{dt}\right|_{t=\nu T-0} = -\Omega_{K,\nu}\,\delta y_i\Big|_{t=\nu T}$$

$$\Omega^2_{K,\nu} = \left(\frac{e^2 N_{e,\nu}}{m\varepsilon_0 D_y^3}\frac{D_y}{c}\right)^2 \frac{96/\pi^4}{\left(1-\bar{b}\right)^2}\sum_{n_y=1}^{\infty}\left[\sum_{n_x=1}^{\infty}\frac{n_y}{n_x}\frac{\sin\left(\frac{\pi}{2}n_x\right)\sin\left(\frac{\pi}{2}n_y\right)}{n_x^2\frac{D_y}{D_x}+n_y^2\frac{D_x}{D_y}}\left[\cos\left(\pi n_x\bar{b}\right)-\cos\left(\pi n_x\right)\right]\right]^2$$

Life-time

✓ *Trapping by the positrons train: qualitative picture*

$$\left.\frac{d\delta y_i}{dt}\right|_{t=\nu T+0} - \left.\frac{d\delta y_i}{dt}\right|_{t=\nu T-0} = -\Omega_{K,\nu}\,\delta y_i\big|_{t=\nu T}$$



Life-time

✓ *Trapping by the positrons train: transfer matrix formulation*

$$\begin{pmatrix} \delta y_i(T-0) \\ \delta \dot{y}_i(T-0) \end{pmatrix} = \begin{bmatrix} \cosh(\Omega_p T) & \Omega_p^{-1}\sinh(\Omega_p T) \\ \Omega_p \sinh(\Omega_p T) & \cosh(\Omega_p T) \end{bmatrix} \begin{pmatrix} \delta y_i(0) \\ \delta \dot{y}_i(0) \end{pmatrix}$$

$$\begin{pmatrix} \delta y_i(t=T+0) \\ \delta \dot{y}_i(t=T+0) \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ -\Omega_{K,\nu} & 1 \end{bmatrix} \begin{pmatrix} \delta y_i(t=T-0) \\ \delta \dot{y}_i(t=T-0) \end{pmatrix}$$

Eigen-value

$$\begin{bmatrix} \cosh(\Omega_p T) & \Omega_p^{-1}\sinh(\Omega_p T) \\ \Omega_p \sinh(\Omega_p T) & \cosh(\Omega_p T) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\Omega_{K,\nu} & 1 \end{bmatrix} = e^{j\omega T} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

✓ *Dispersion relation*

Kick

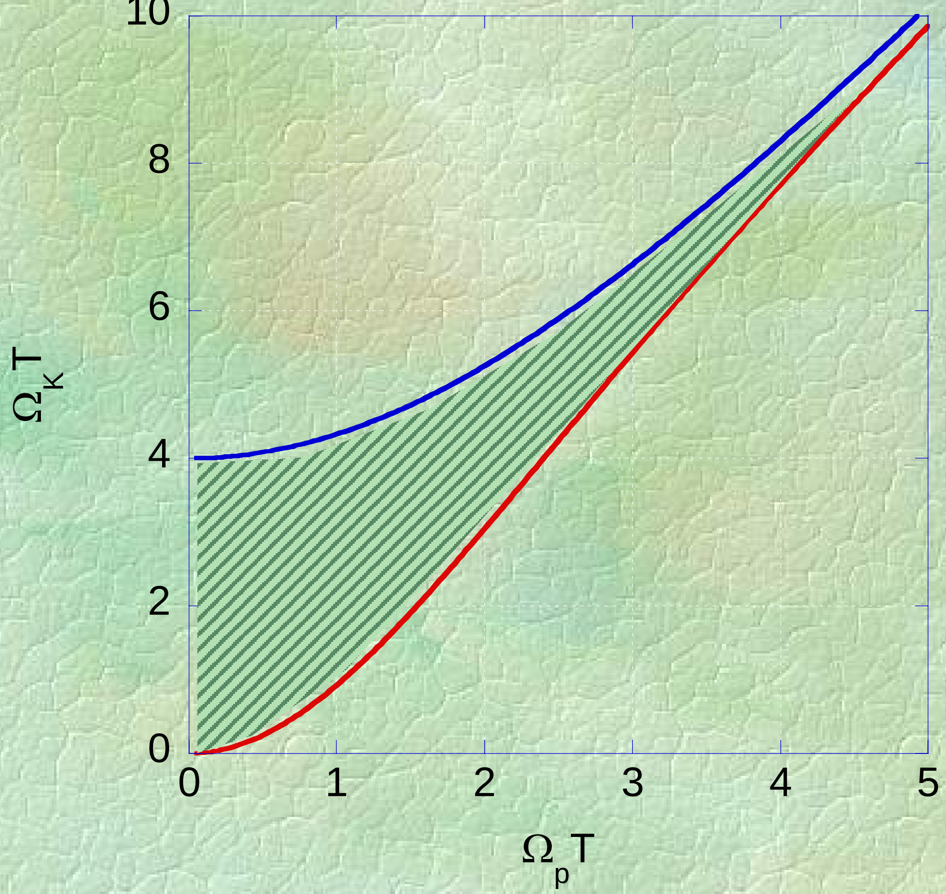
T- determines the scale

$$\boxed{\cos(\omega T) = \cosh(\Omega_p T) - (\Omega_{K,\nu} T / 2\Omega_p T)\sinh(\Omega_p T)}$$

Life-time

✓ *Trapping by the positrons train: Dispersion*

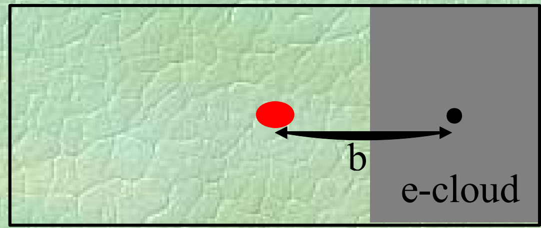
$$\left|\cos\left(\omega T\right)\right| < 1 \Rightarrow 2\Omega_p T \tanh\left(\Omega_p T/2\right) < \Omega_{K,\nu} T < \frac{2\Omega_p T}{\tanh\left(\Omega_p T/2\right)}$$



✓ *For low values trapping is feasible*

✓ *At high values the constraint is very stringent*

Life-time



✓ *Trapping by the positrons train: Dispersion*

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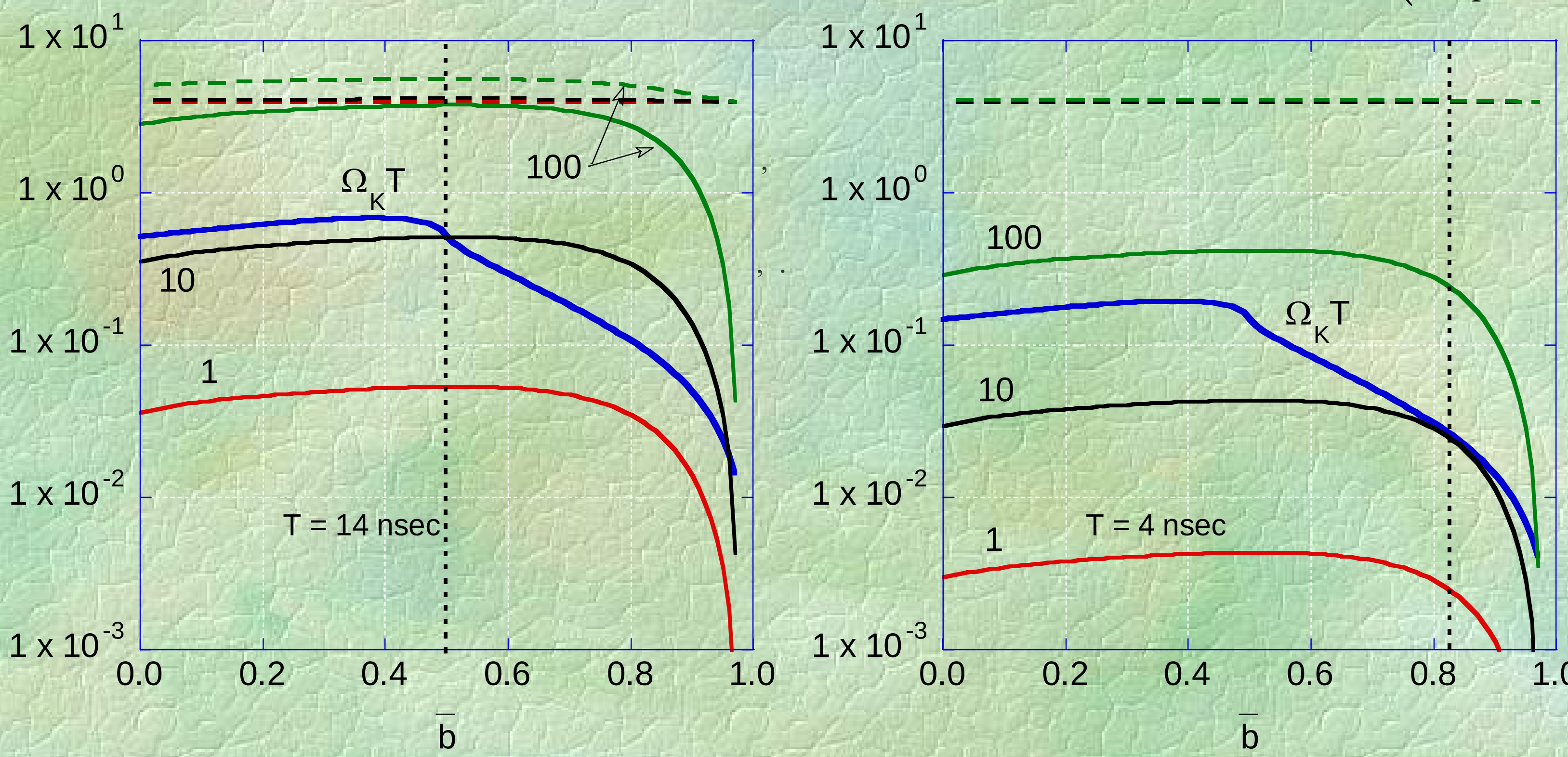
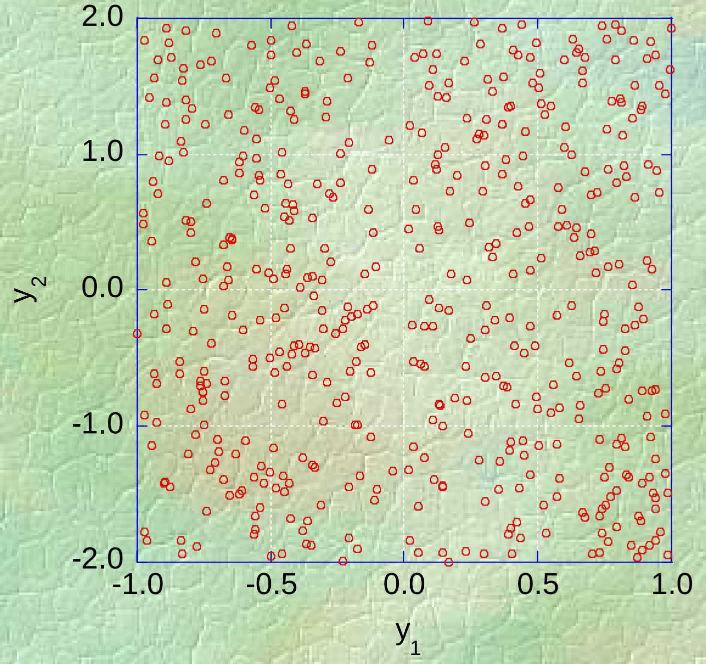
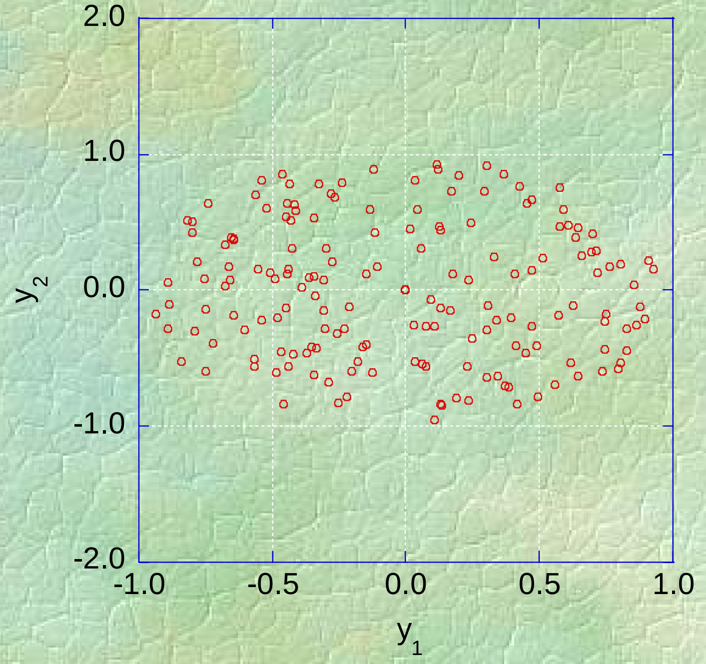


$N_{pos} = 10^{10}$ $\quad n_{ec}[m^{-3}] \times 10^{-11} = 1, 10 \text{ and } 100$

Life-time

✓ ***Trapping*** ***by the positrons train: Phase-space***





Only a fraction of the electrons are being trapped

$$\bar{y}_1^2 + \bar{y}_2^2 \leq 1 \qquad E \leq E_p \left[1 - \bar{y}_1^2\right] \leq E_p \equiv E_{\max,\mathrm{ps}}$$

Life-time

*Train of **positron** bunches*

$$\tau = \underbrace{\int_0^{E_{\max,ps}} dE\, f_{ec}^{(p)}(E)\frac{1}{2}NT}_{Trapped} + \underbrace{\int_{E_{\max,ps}}^{\infty} dE\, f_{ec}^{(p)}(E)\tau'(E)}_{Free}$$

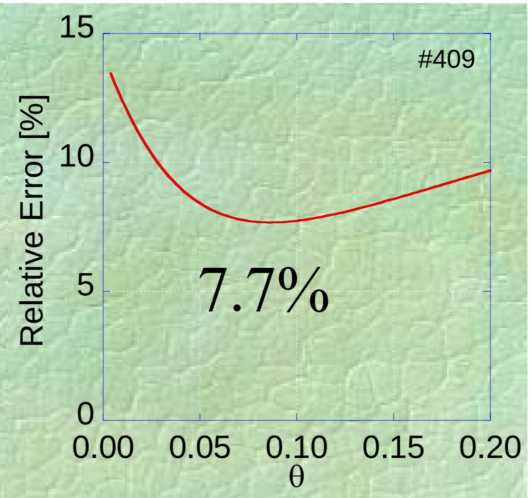
$$\simeq \underbrace{\frac{1}{2}NT\frac{E_{\max,ps}}{E_0}}_{Trapped} + \underbrace{\frac{1}{E_0 - E_{\max,ps}}\int_{E_{\max,ps}}^{E_0} dE\,\tau'(E)\left(1-\frac{E_{\max,ps}}{E_0}\right)}_{Free}$$

*Train of **electron** bunches*

$$\tau = \int_0^{\infty} dE\, f_{ec}^{(e)}(E)\tau'(E) \simeq \frac{1}{E_0}\int_0^{E_0} dE\,\tau'(E)$$

Theory and Experiment

$$\delta Q_\mu [\%] \simeq \bar{q} \sum_{\nu=1}^{N} W_{\mu,\nu} \exp\left[-\frac{T}{\tau}(\mu-\nu)\right]$$

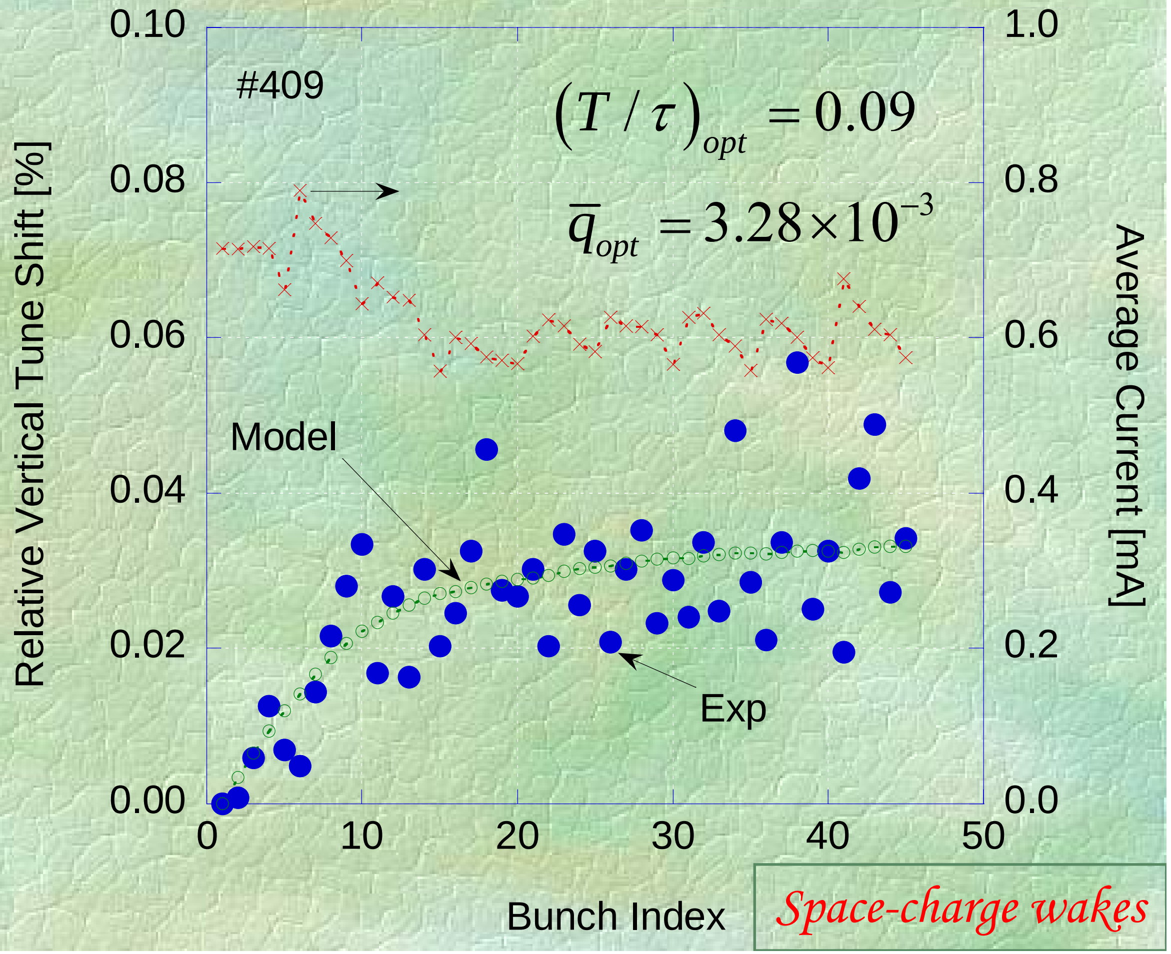


$$\bar{q} \equiv 50 \frac{\beta_r^2 r_e}{\gamma} \frac{D_x}{D_y} f_E(\bar{b}) n_{nb}$$

$$W_{\mu,\nu} = \frac{I_\nu}{\langle I \rangle} h(\mu-\nu-0)$$

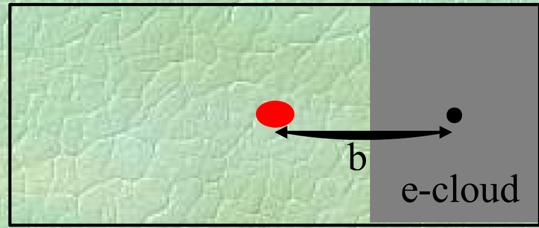
- ✓ Av. current 0.63[mA]
- ✓ Vert. feedback=off
- ✓ Unknowns:

<E>, b, τ, n_{ec},



Space-charge wakes

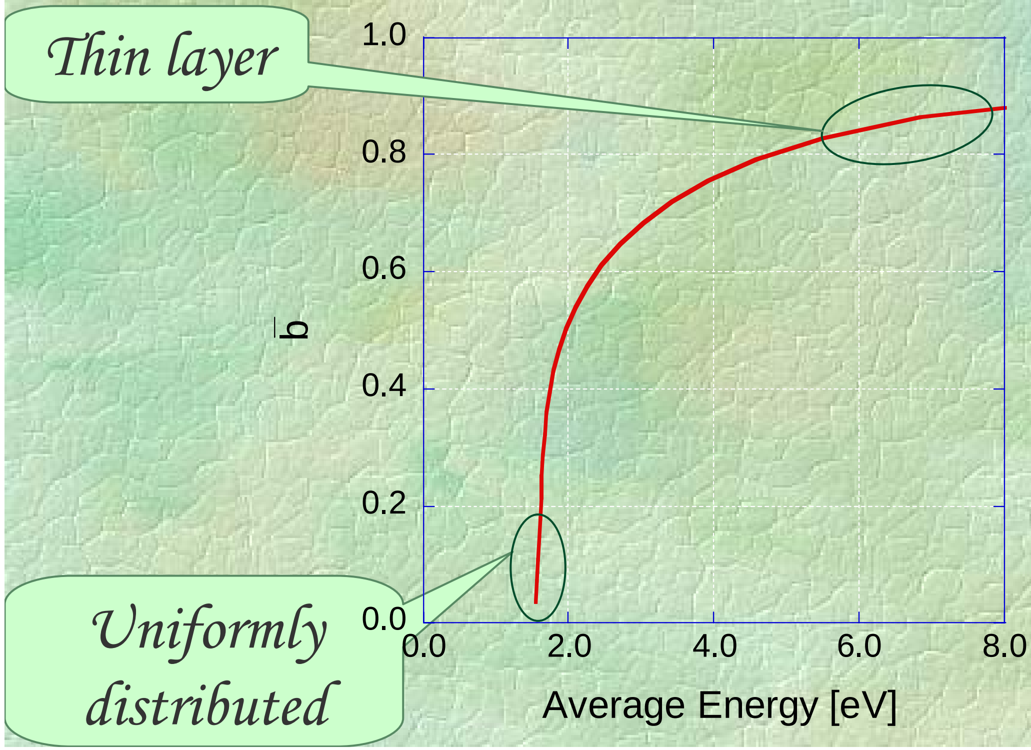
Theory and Experiment

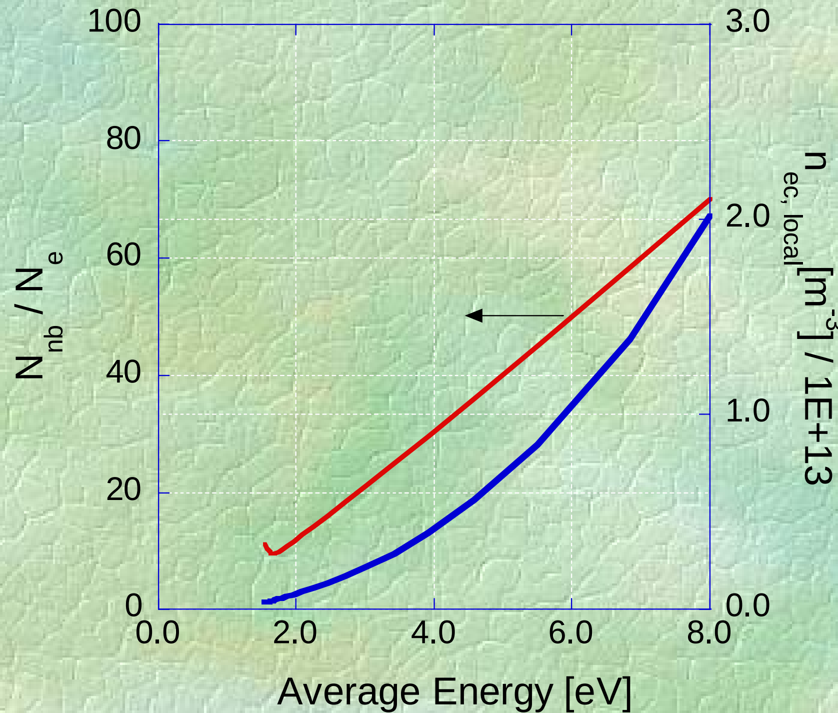


a) $(T/\tau)_{opt} = 0.09 \Rightarrow \tau_{opt} \simeq 155n\sec \Rightarrow E_{eff} = 0.6[eV]$

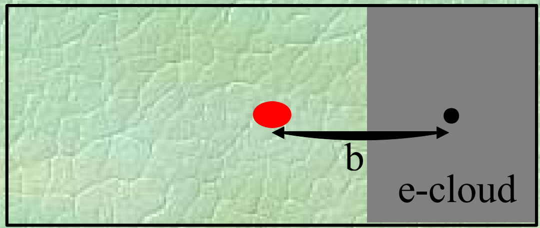
b) $\bar{q}_{opt} = 3.28\times10^{-3} = 50\dfrac{\beta_r^2 r_e}{\gamma}\dfrac{D_x}{D_y} f_E\left(\bar{b}\right)n_{nb} \Rightarrow f_E\left(\bar{b}\right)n_{nb} = 2\times10^{11}\left[m^{-3}\right]$

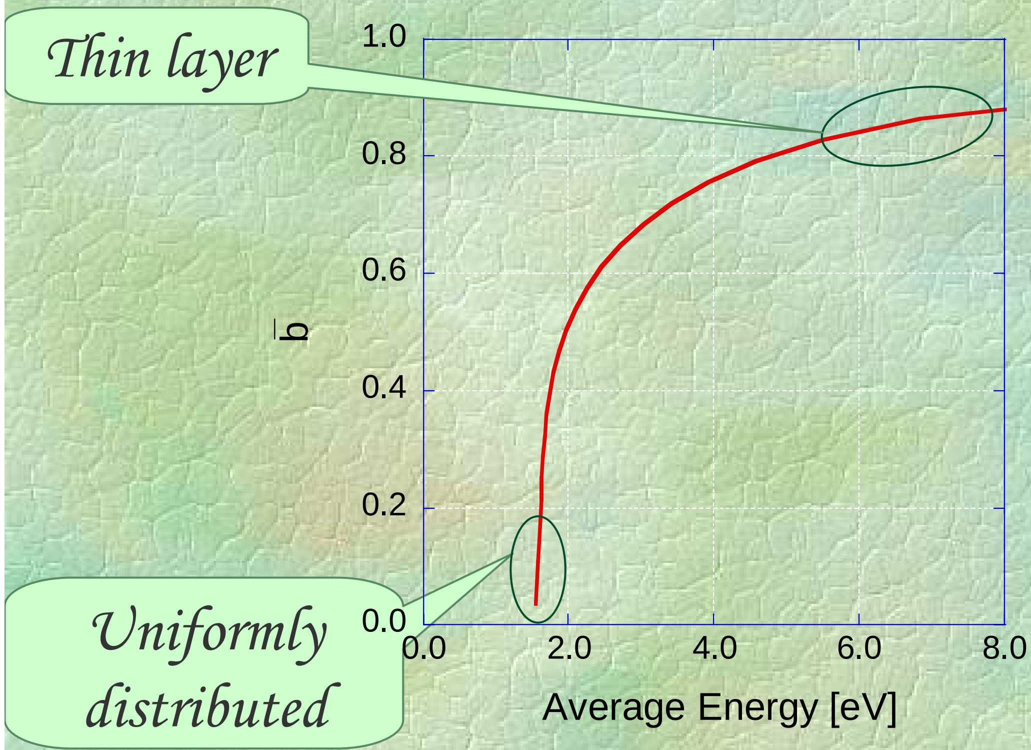
c) $n_{\rm ec} \simeq n_{\rm nb}\tau / T = n_{\rm nb} / \theta_{opt} \simeq 11 n_{\rm nb}$

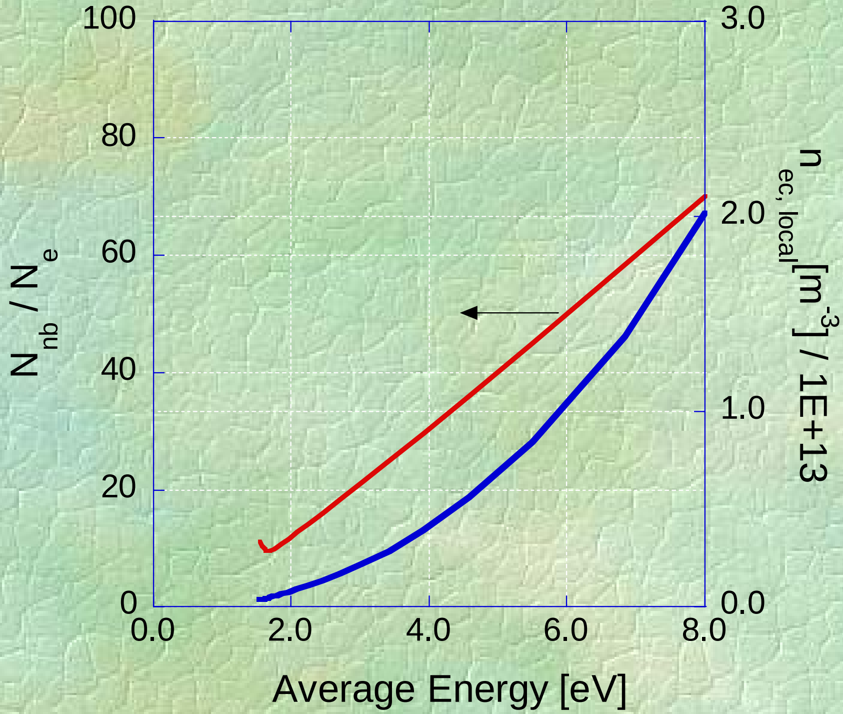




Theory and Experiment







- ✓ *With exception of the low-energy end ,the model indicates that number of "new-borns" per positron, is **linear** with the average energy of the cloud*
- ✓ *Quadratic **local density** on the average energy ; less than 10^{11} e/m*

Theory and Experiment

- *The **photo-electrons** can account for only 1 new-born out of more than 10 (@2eV).*

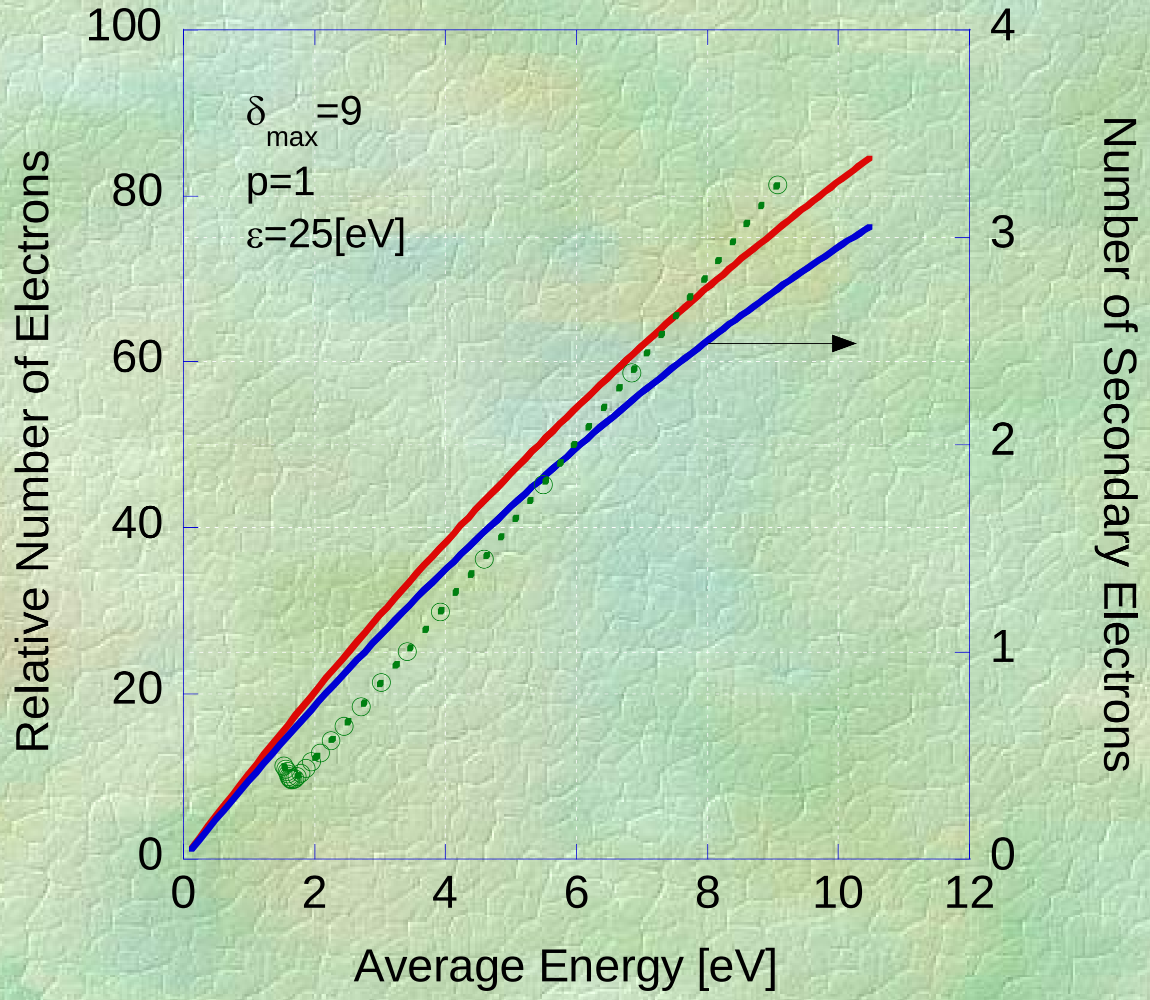
$$\frac{N_{\text{pe}}}{N_e} = 130 \times E_k[GeV]\,\overline{\delta}_{\text{pe}}\left(8\frac{\langle E\rangle}{E_{\text{cr}}}\right) \sim 0.94 @ 2eV$$

- *Secondary electrons contribute the remainder .*

$$N_{\text{se}} = 130 \times E_k[GeV]\,\overline{\delta}_{\text{pe}} N_e \int_0^\infty dE\, f_{\text{se}}(E)$$

$$\langle E\rangle = \langle E\rangle_{\text{pe}} \frac{N_{\text{pe}}}{N_{\text{pe}} + N_{\text{se}}} + \langle E\rangle_{\text{se}} \frac{N_{\text{se}}}{N_{\text{pe}} + N_{\text{se}}}$$

Theory and Experiment



- ✓ *If only SE are considered, the max SEY is high alluding to presence of* Al_2O_3
- ✓ *Or that other (linear) mechanisms play an important role ("stray" or bound)*

Summary & Conclusions

- ✓ The **vertical tune-shift** is determined by the **average density as experienced by each bunch**
- ✓ **Build-up** of this average density is determined by two parameters: "new-born" & life-time ("charging-up": linear rise & equilibrium)
- ✓ **"New-born"** include:
  - photo electrons
  - secondary electrons
  - stray electrons
  - previously bound electrons (ionization)
- ✓ **Life-time** in case of a train of positrons has two contributions. Of:
  - fast electrons ($E>E_p$) that traverse the chamber
    - space-charge of the cloud
    - image-charge at the walls
  - trapped electrons ($E<E_p$) resembling ion trapping

Summary & Conclusions

- ✓ **Least-mean square fit** (model & experimental data) provides an estimate of the cloud's parameters: density, geometry and life-time.
- ✓ The **long life-time** of the cloud is not consistent with a multi-pactoring which requires energies of the order of 300[eV] or higher.
- ✓ Average energy of electrons in the cloud is a few electron volts.
- ✓ **Space-charge** waves may develop along this cloud and explain the "fluctuations" when the cloud reaches equilibrium

Acknowledgement

- ✓ Introducing me to the subject: Maury Tigner, Mark Palmer & David Rice
- ✓ Great support with elucidation of the experimental data: Jerry Codner, Eugene Tanke and Robert Holtzapple
- ✓ Jim Crittenden was very helpful providing me with the detailed lattice data with and w/o wigglers.
- ✓ Kathey Harkay and John Flanagan for their input re this model.
- ✓ DoE and NSF