Tracking code for microwave instability

S. Heifets, SLAC

ILC Damping Rings R&D Workshop
Cornell University

September, 2006

To study microwave instability the tracking code is developed. For bench marking, results are compared with Oide-Yokoya results for broad-band $Q = 1$ impedance. Results suggests two possible mechanisms determining the threshold of instability.

Tracking Map

The Fokker-Plank equation

$$\frac{\partial \rho}{\partial \tau} + \{H(p, x, \tau), \rho\} = \Gamma \frac{\partial}{\partial p} \left(\frac{\partial \rho}{\partial p} + p\rho\right),$$

where

$$x = \frac{z}{\sigma_0}, \quad p = -\frac{\delta}{\delta_0}, \quad \tau = \omega_s t$$

$\Gamma = (\omega_s t_d)^{-1}$, t_d is SR damping time, $H(p, x, \tau)$ is the Hamiltonian
can be replaced by the map with time step $\delta\tau$

$$
\begin{aligned}
\bar{p}_i &= p_i - \delta\tau\Gamma p_i + \delta\tau\{-x_i \\
&+\lambda\sigma_0 \int dx' f(x',\tau) W[\sigma_0(x' - x_i(\tau)]\} + a\,\xi, \\
\bar{x}_i &= x_i + \delta\tau\,\bar{p}_i.
\end{aligned}
$$

Here,

$$
\lambda = \frac{N_b r_e}{\gamma\alpha\delta_0^2 C}, \quad a = \sqrt{6\Gamma\delta\tau}.
$$

ξ_k is a random variable uniformly distributed in the interval
$-1 < \xi < 1$,
$f(x,\tau) = \int \rho(x,p,\tau) dp$
is defined on each time step by interpolation on a mesh following the bunch centroid at high currents.

Code

The map is used for tracking. Approach was used before to study microwave instability by K. Bane and by Oide and Yokoya.

The FORTRAN works in steps:

a) Haissinki distribution $\rho(x, p)$ using Newton's iteration method suggested by R. Warnock. Below threshold, rms $\delta = 1$.

b) Coordinates $\{x_i, p_i\}$ of M macro-particles are generated with the probability given by $\rho(x_i, p_i)$.

c) Tracking starts calculating rms energy spread δ on each step.

d) Stop when $\delta > \delta_{max} = 1.1$ or number of steps exceeds n_m. In the first case, the current is taken as the threshold of instability. In the later case, the process is repeated with the higher value of current.

Wake

Results of simulations described below are obtained, for bench mark purpose, with the BB $Q = 1$ resonator wake with 2 parameters

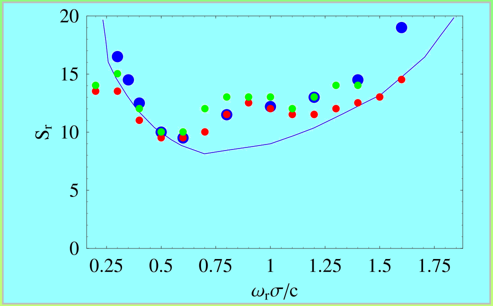
$$x_r = \frac{\omega_r \sigma_0}{c}, \quad S_r = \frac{I_{bunch} r_e}{ec} \frac{4\pi\omega_r}{\gamma\omega_s\delta_0} \frac{R_s}{Z_0},$$

Checking code

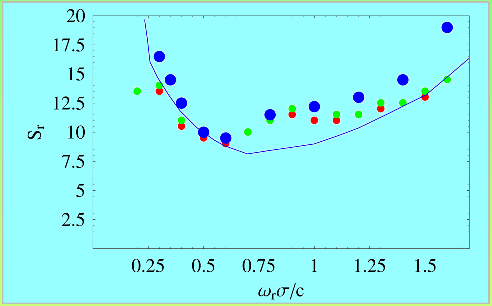
a) The results were checked with different time steps. For $\delta\tau < 0.5$ calculations are stable and are not sensitive to $\delta\tau$.
b) Calculations give results independent on the number of random kicks per time step.

Calculations

Normally, we use
$\delta\tau = 0.125$ (50 steps per synchrotron period) and one random kick per time step,
the damping time $\Gamma = 1.\,10^{-2}$,
the number of time steps 8000 corresponds to 10 damping times.
Mesh: 101 bins mesh with the mesh size $-10 < x < 10$.
CPU time $1.1\,\mu s$ per step/per particle on 2.4 GHz PC.



The threshold of the microwave instability for $Q = 1$ impedance. Results of Oide-Yokoya are shown by blue line and their tracking results by large blue dots. Our calculations are shown by small dots: for 10^4 macro-particles in green, and for 5.10^4 macro-particles in red.

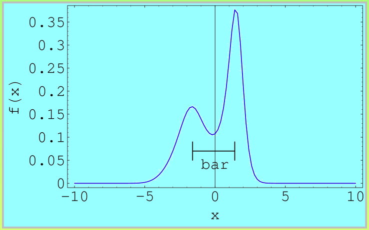


Results of Oide-Yokoya are shown in blue. Our calculations are shown by small dots for two damping times: $\Gamma = 10^{-2}$ (red), and $\Gamma = (1/3)\, 10^{-2}$ (green).

The main uncertainty in the threshold comes from the arbitrariness of δ_{max} because the energy spread grows slowly close to the threshold of instability and δ, calculated by tracking, fluctuates. This problem can be elevated using large number of macro-particles what requires larger CPU time. In simulations we use $\delta_{max} = 1.1$ (Oide and Yokoya used $\delta_{max} = 1.05$).

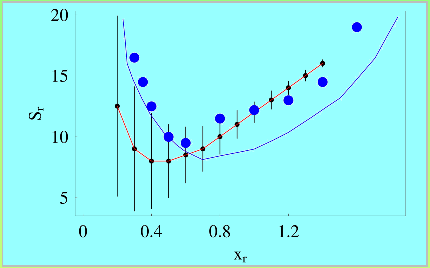
Tracking results deviate from the curve predicted by Oide-Yokoya at large $x_r > 0.5$.

It was suggested before (P. Wilson, private communication) that the threshold of the microwave instability is related to appearance of a new maximum of the Haissinski steady-state distribution.



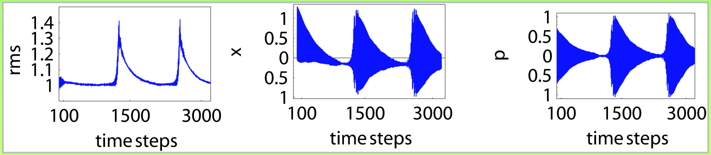
Example of the Haissinski distribution. $x_r = 0.8$, $S_r = 14.85$. The bar shows the distance between maxima.

This Fig. shows that the conjecture seems to be valid at $x_r > 0.5$ suggesting two mechanisms of instability.

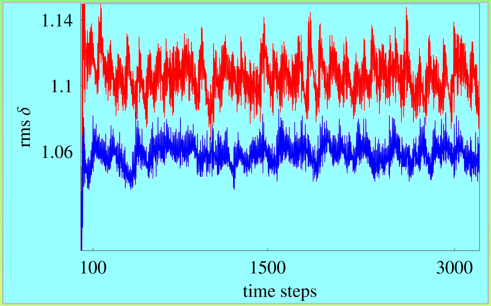


Comparison of the threshold of the microwave instability (blue line) with the threshold of two-minima Haissinski potential well. The total length of the bar is the distance between minima (in x). Both curves tend to coincide at $x_r > 0.5$ while for smaller x_r the second minimum is on the far tail of the distribution and does not cause the instability.

Tracking allows not only to define the threshold but also to study the dynamics of instability above the threshold. Figs. below give examples of the beam dynamics above the threshold for small $x_r = 0.3$ and large $x_r = 0.8$. More results are reported in S. H., SLAC-PUB-12122.



Dynamics of the microwave instability above threshold for small x_r: $x_r = 0.3$, $S_r = 13.84$, $\Gamma = 1.0\,10^{-2}$. Total number of time steps 16000 with $\delta\tau = 0.125$ (each 5-th step is recorded). The total time of tracking $t/t_d = 10$.



Time dependence of the rms energy spread above the threshold of microwave instability for large x_r. $S_r = 11.8$ (blue), $S_r = 13.02$ (red), in both cases $x_r = 0.8$.