# A new solver for microwave instability

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# Motivation

- According to Boussard criterion and the wake model in Configuration Studies, ILCDR is unstable with respect to the longitudinal single bunch microwave instability. However, both the impedance model and the stability criterion are very crude.
- We expect that we will have a better impedance model at some time in future. By that time we will need reliable codes for the microwave stability analysis. In addition to computing the threshold of the instability, we would like to understand the sensitivity of the results to possible uncertainties between the model and the real machine
- It was proposed to the ICSDR mailing list to benchmark the existing codes on a set of theoretical problems. There was also a discussion to benchmark the codes against experimental observations.

# Microwave instability

There are several approaches to calculate the longitudinal stability of the beam. This is the stability of a *Haïssinski* equilibrium.

1. Boussard criterion—crude, order of magnitude estimation

$$\langle Z/n \rangle \approx \gamma \alpha \sigma_\delta^2 \sigma_z / N r_e$$

2. Particle tracking with wakefields—fast, but may be noisy
3. Solving of linearized Vlasov integral equation in frequency domain (Oide&Yokoya)—has a lot of neutrally stable spurious modes
4. Solving full Vlasov-FP equation in time domain—slow, but accurate

Any of those methods can be used to predict the beam stability. I developed a new code where a linearized Vlasov equation is solved in time domain. It was expected that the code would have advantages of the full Vlasov code but faster. The code is implemented in Mathematica—easy to modify and expand.

# Microwave instability in ILCDR

Some thoughts about organizing the study of instabilities for the ILCDR.

- We cannot wait until we get a reasonably accurate wakefield for the ring after vacuum chamber design is finished and the wakes are calculated. There will be always uncertainty in the wake.
- It makes sense to debug and benchmark our theoretical and simulation tools, make comparison with the experiment. It is important to have a setup that allows a quick recalculation of the ring stability properties when changes to the vacuum chamber are made.
- I think it is useful to explore various wakefields beforehand to get a feeling of their stability properties.

# Some representative wakefields

1. Broadband resonator impedance wakefield has 3 parameters: $R$, $Q$, and $\omega_0$

$$w(s) = \frac{\omega_0 R}{Q} e^{-\omega_0 s/2Qc} \left( \cos(\omega_1 s/c) - \frac{\sin(\omega_1 s/c)}{\sqrt{4Q^2 - 1}} \right)$$

   Microwave instability was studied for $Q = 1$ by Oide&Yokoya.

2. Resistive wake—beam is always unstable

$$w(s) = cR\delta(s)$$

3. Inductive wake—beam is always stable

$$w(s) = L\delta'(s)$$

4. Free space CSR wake—there is a threshold

$$w_{\mathrm{CSR}}(s) = -\frac{Z_0 Cc}{2 \cdot 3^{4/3} \pi \rho^{2/3}} \frac{1}{(-s)^{4/3}} \, ,$$

We can add those wakes with weights to model more complex wake functions.

# Linear stability problem

The beam longitudinal dynamics is described by a distribution function $\psi(t, z, \delta)$. First, we find the equilibrium $\psi_0(z, \delta)$ by solving Haïssinski equation. We then linearize the Vlasov equation, assuming $\psi(t, z, \delta) = \psi_0(z, \delta) + \psi_1(t, z, \delta)$, $|\psi_1| \ll |\psi_0|$

$$\frac{\partial \psi_1}{\partial t} - c\eta\delta \frac{\partial \psi_1}{\partial z} + K_0(z) \frac{\partial \psi_1}{\partial \delta} + K_1(t, z) \frac{\partial \psi_0}{\partial \delta} = RHS$$

$$K_1(z, t) = -\frac{cr_e}{\gamma} \int_{-\infty}^{\infty} dz' d\delta \psi_1(t, z', \delta) w(z' - z)$$

$$K_0(z) = \frac{\omega_{s0}^2}{\eta c} z - \frac{cr_e}{\gamma} \int_{-\infty}^{\infty} dz' d\delta \psi_0(z', \delta) w(z' - z)$$

where is $\eta$ the slip factor. RHS takes into account synchrotron radiation damping and quantum diffusion.

# Linear stability problem

In my code, this equation is solved numerically on a mesh in $\zeta$-$p$ space ($p = -\delta/\sigma_\delta$, $\zeta = z/\sigma_z$), starting from a randomly generated initial $\psi_1$. Typically the size of the mesh $150\times150$ ($250\times250$ in tests). If the system is unstable, after long enough time, the evolution is dominated by the fastest growing mode. The growth rate $\gamma$ can be found numerically, as well as the phase portrait of this unstable mode. Typical time step $\Delta t\omega_{s0} = 0.01$.

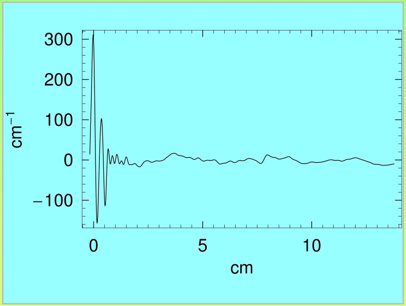
Radiation damping and quantum diffusion is neglected, RHS $= 0$. This can be corrected by subtraction of the radiation damping rate from the growth rate of the instability.

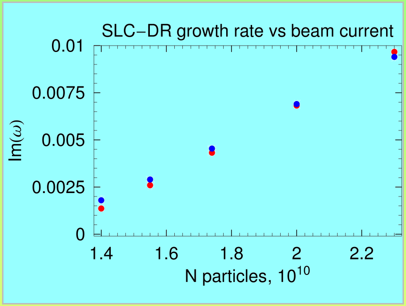
The algorithm is implemented as a Mathematica code. The wake can be input as a Mathematica function and can include predefined resistive, inductive, and the CSR wake. The code is reasonably fast.

Movie: Start from noise

# Testing code on SLCDR wake

The code has been tested on the SLCDR wake (K. Bane), previously studied by several other codes [Warnock et al., EPAC 2004]



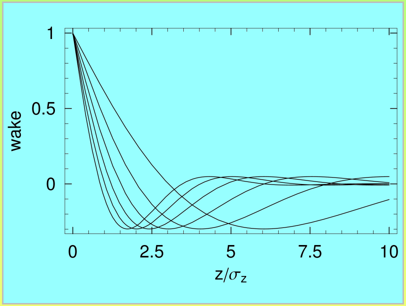




Wakefield for the SLCDR and comparison of the new code (blue dots) with the published results (red dots). The growth rate $\mathrm{Im}\,\omega$ is in units $\omega_{s0}$. The radiation damping in the ring is $\gamma \approx 0.001\omega_s$.

# Broadband resonant impedance

Broadband resonant impedance has been studied in the past using an eigenmode solver by Oide and Yokoya (1990) and by Mosnier (1999). The threshold depends on $Q$ and $\omega_0\sigma_z/c$. The dimensionless current is

$$S = \frac{2N}{\gamma\nu_{s0}\sigma_\delta}\frac{\omega_0 r_e}{c}\frac{R}{QZ_0}$$

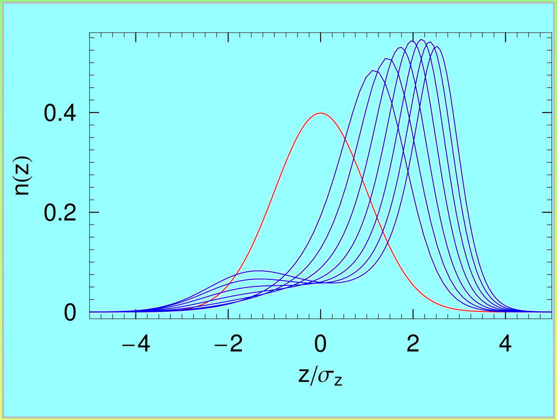



The stability threshold is defined by $S_{\rm th} = S(Q, \omega_0\sigma_z/c)$. Most of the results refer to $Q = 1$.

Wakes for $Q = 1$, $\omega_0\sigma_z/c$ varies from 0.4 to 1.5.

# Broadband resonant impedance

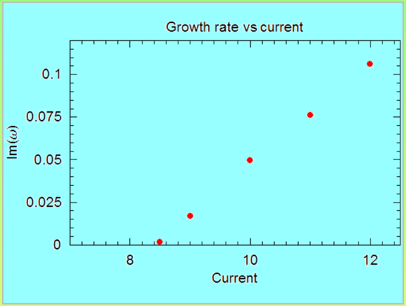
Haïssinski equilibria for $Q = 1$, $\omega_0 \sigma_z / c = 0.6$, $S = 4, \ldots, 10$

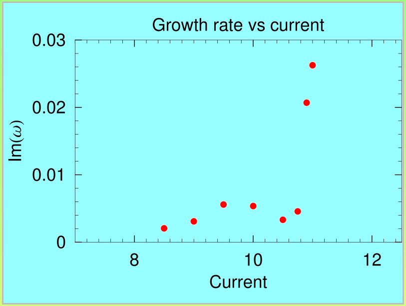



Double-hump distribution requires a special treatment in the Oide&Yokoya approach.

# Comparison with Oide&Yokoya results

Threshold is found by interpolating the growth rate to zero as a function of current.

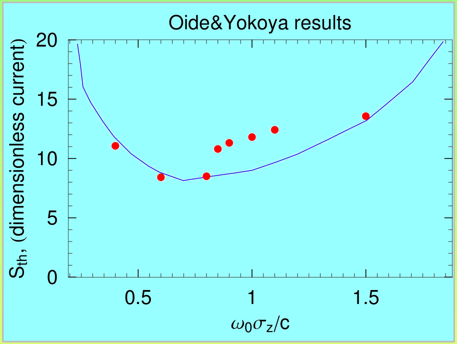



Growth rate vs $S$ for $\omega_0\sigma_z/c = 0.6$ (left) and $\omega_0\sigma_z/c = 0.85$ (right), $Q = 1$.
For the 6 km ILCDR, $\gamma/\omega_{s0} \approx 0.009$.
Movie.

# Comparison with Oide&Yokoya results

The stability threshold is defined by $S_{\mathrm{th}} = S(Q, \omega_0 \sigma_z / c)$

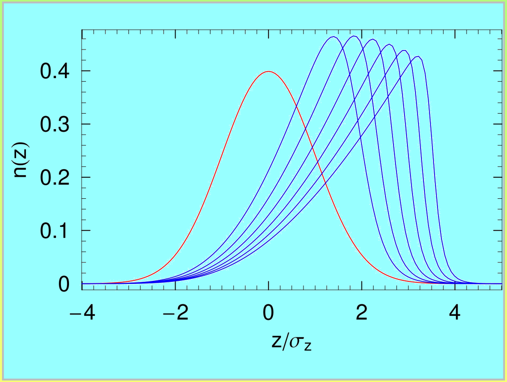



Threshold current as a function of $\omega_0$, $Q = 1$.

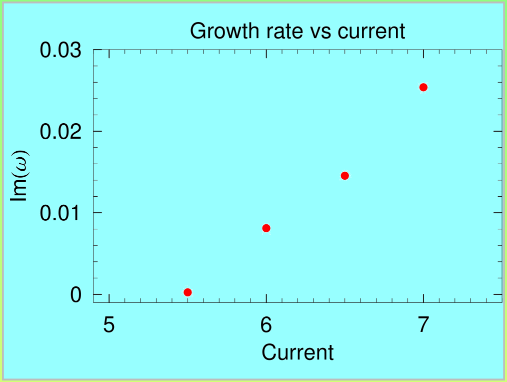
# CSR impedance

CSR impedance in free space has a singularity at the origin. CSR equilibrium and stability depends only on one dimensionless parameter. Haïssinski equilibria for , $S = 3, \ldots, 8$

$$S = \frac{N}{3^{1/3}\pi\gamma\nu_s\sigma_\delta}\frac{Cr_e}{\rho^{2/3}\sigma_z^{4/3}} .$$



# Microwave instability with CSR impedance




The microwave instability threshold for the CSR impedance is

$$S = 5.5$$

Movie.

# Conclusion

- A new code is developed that solves linearized Vlasov equation. The code is implemented as a Mathematica notebook and is flexible enough to use various wakefields including resistive, inductive and CSR wakes.
- Test runs showed agreement with the previous studies of the microwave instability for the SLCDR. Runs with broadband resonant impedances confirm old results of Oide and Yokoya.
- The threshold for the CSR instability was numerically found using the new code.