ILC Damping Rings Mini-Workshop

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The Effect of Beta Function Variation on Wakefield Coupled Bunches

Kai Hock and Andy Wolski

University of Liverpool and the Cockcroft Institute

Introduction

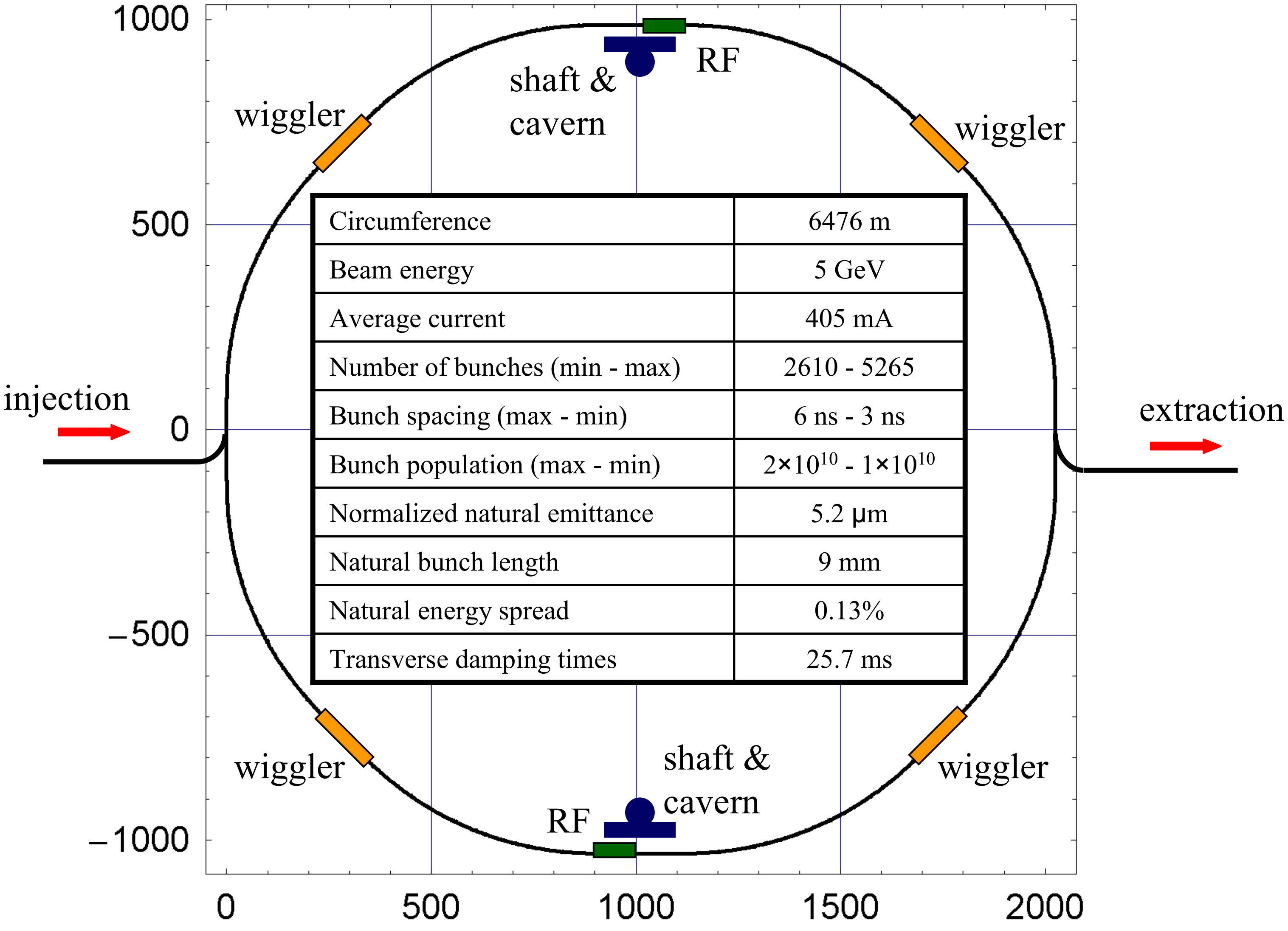
To study the transverse instability of bunches coupled by the resistive wall wakefield in the ILC damping ring.

1. Time domain simulation for OCS6 damping ring
   . Standard model and analytical results
   . Simulation method on real lattice

2. Unexpected behavior on the real lattice
   . Deviations from model in a simple lattice
   . Comparison with effect of HOM, uneven fill

3. Transient effects during injection and extraction
   . Fill patterns, feedback system, emittance, NEG coating, …
   . Convergence issues

The ILC damping rings baseline configuration



Circumference	6476 m
Beam energy	5 GeV
Average current	405 mA
Number of bunches (min - max)	2610 - 5265
Bunch spacing (max - min)	6 ns - 3 ns
Bunch population (max - min)	2×10^{10} - 1×10^{10}
Normalized natural emittance	5.2 μm
Natural bunch length	9 mm
Natural energy spread	0.13%
Transverse damping times	25.7 ms

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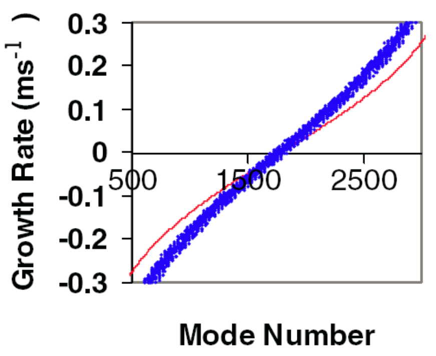
Coupled bunch instabilities

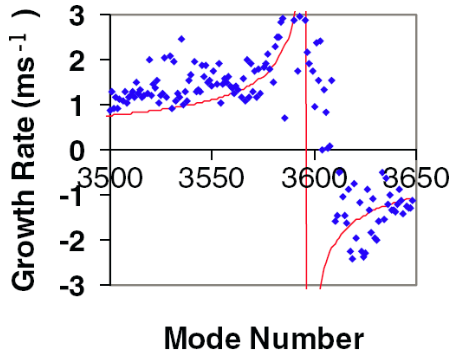
Long-range wake fields in the damping rings are of concern for two reasons:

- Initial estimates based on resistive-wall wake fields indicate coupled-bunch instability growth rates that could be challenging to deal with.
- The large jitter of injected bunches could couple through the wake fields to damped bunches awaiting extraction, leading to bunch-to-bunch jitter in the extracted beam that exceeds specifications.

The stability of the beam extracted from the damping rings is critical for the performance of the ILC, so we are therefore taking a careful look at the effects of long-range wake fields.

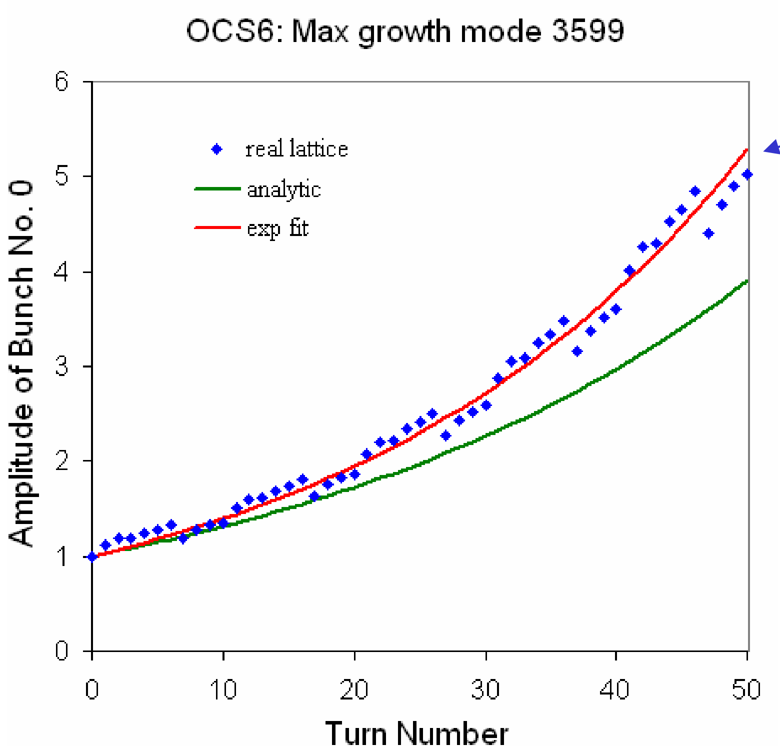
Generally, time-domain simulations confirm the growth rates expected from analytical estimates…





Initial growth rates from simulation using real beta function.

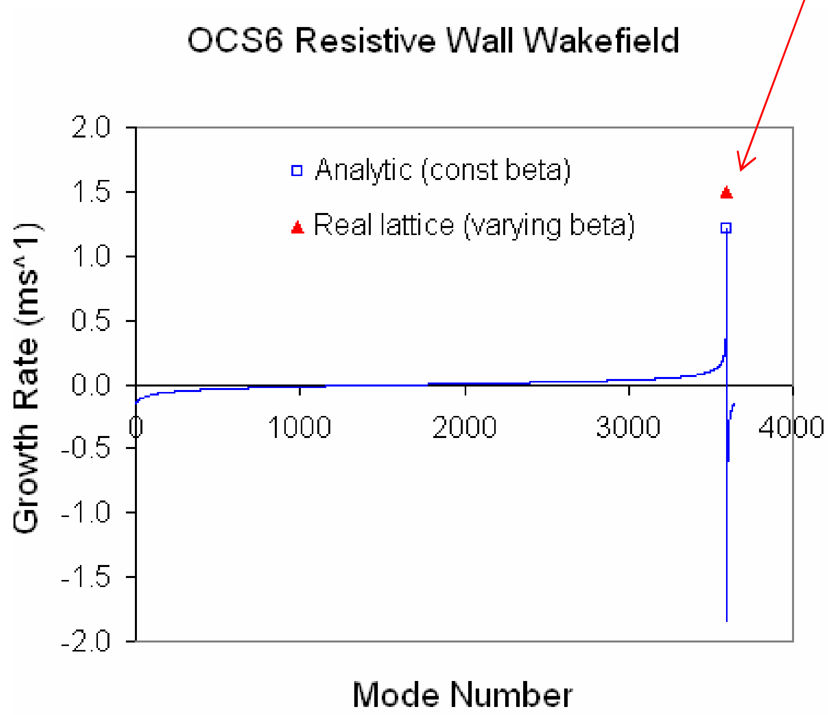
OCS6 Growth Rate



Max. growth rate = 1.49 ms^{-1}

Growth time = 670 μs or 30 turns

23% higher



Electrons / bunch	2×10^{10}
Al conductivity	$3.2 \times 10^{17}\ s^{-1}$
Beam pipe radius	*2 cm*
Vertical tune	*49.31*
Bunch Number	*3649*

Transverse Resistive Wall Wakefield Coupled Bunches

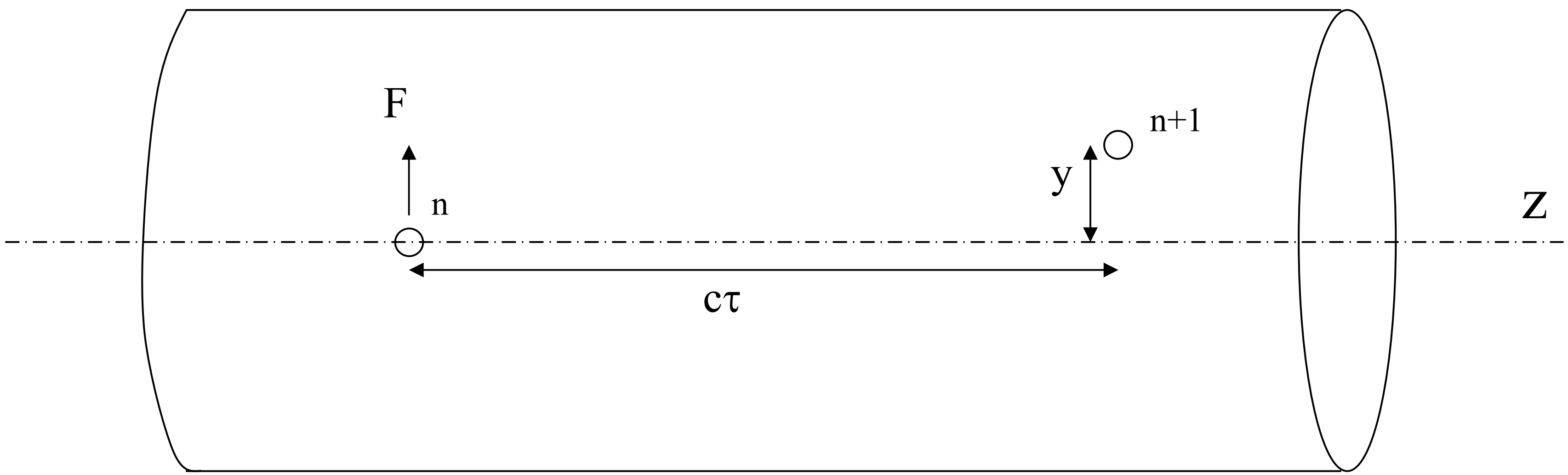
Results so far

- Including the realistic variation in the beta function results in higher growth rates even in a simple lattice, compared to analytic average beta function results.
- Uneven fill pattern, HOM have relatively small effects in the OCS6 damping ring.

Ongoing

- To simulate transient effects on extracted bunches, due to jitter of injected bunches.

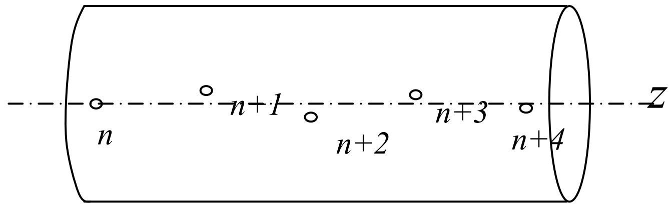
Resistive wall wakefield



Force on bunch n: $F \sim W_1(-c\tau) y_{n+1}$

Assume $W_1(z) \sim \frac{1}{\sqrt{|z|}}$ (Chao 1993)

Standard model & analytic results



Assume constant beta function:

$$\ddot{y}_n(t) + \omega_\beta^2 y_n(t) \quad \sim \quad W_1(-c\tau) y_{n+1}(t-\tau) + W_1(-2c\tau) y_{n+2}(t-2\tau) + W_1(-3c\tau) y_{n+3}(t-3\tau) + \ldots$$

Betatron oscillation *Wakefield sum*

Obtain eigenmodes: $$\tilde{y}_\mu(t) = \sum_{m=0}^{M-1} y_m(t) e^{-i\frac{2\pi m\mu}{M}}$$ (Fourier modes)

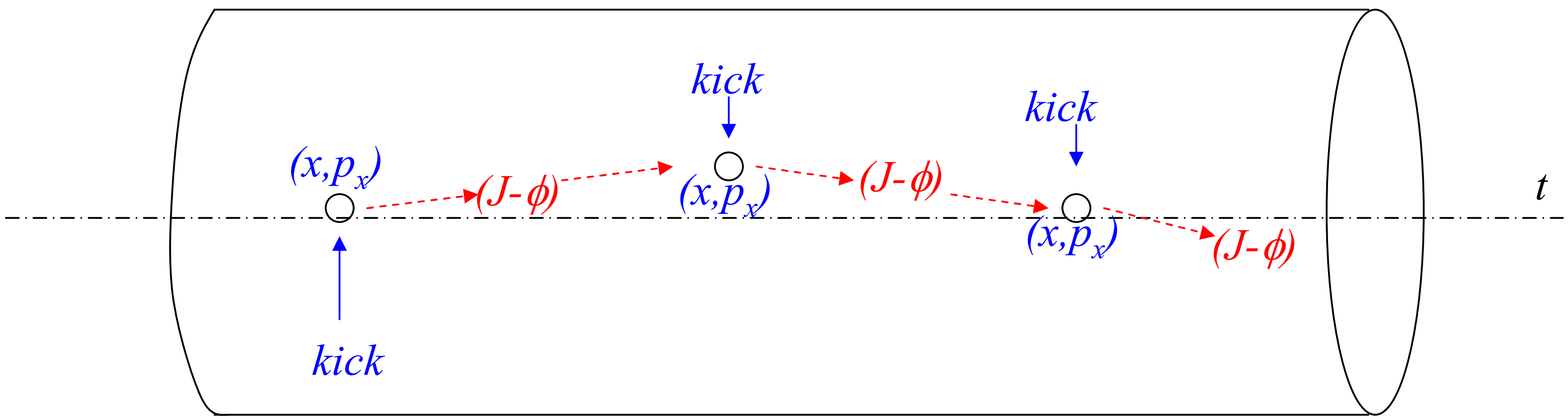
Derive growth rates: $$\Omega_\mu - \omega_\beta = -i\frac{MNr_0 c}{2\gamma T_0^2 \omega_\beta} \sum_{p=-\infty}^{\infty} Z_1^\perp[\omega_\beta + (pM+\mu)\omega_0]$$

$$\frac{1}{\tau_\mu} = \mathrm{Im}\,\Omega_\mu$$ (widely used)

Simulation Method

Should not integrate equation of motion directly as this is assumes constant beta function:

1. At each time step, add kick to momentum from wakefield.
2. Transform to action angle variables (J-ϕ), phase advance 1 step.
3. Transform back to Cartesian (x,p_x), add kick, repeat.



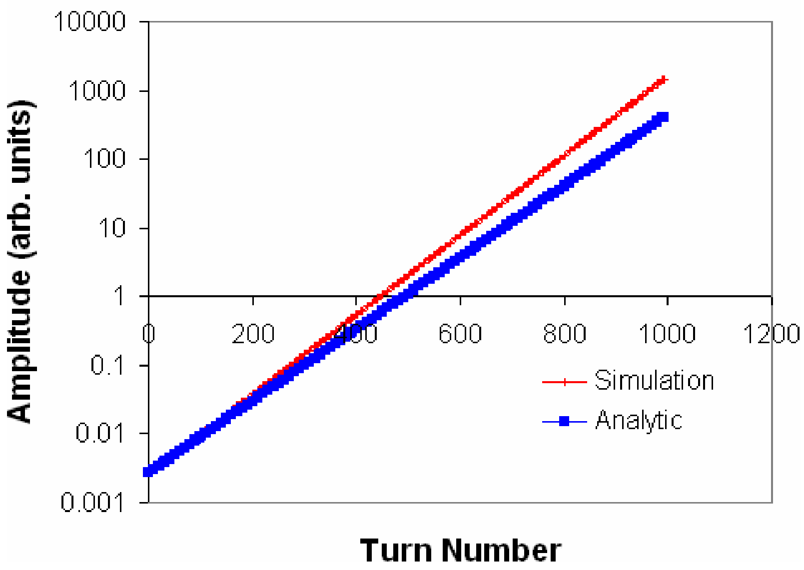
The actual (varying) beta function is used in the phase advance.

Simple Lattice – 10 FODO Cells

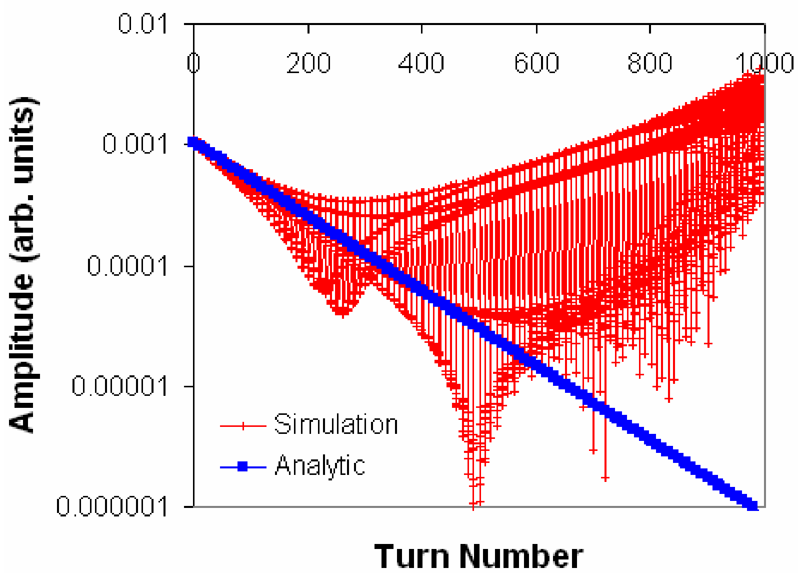
Variation in beta function causes coupling of multi-bunch modes. As a result …

Max. growth rate is larger than analytic result for constant beta.

Decay modes can grow.



(a)



(b)

Figure 24. Amplitudes of (a) mode 2 and (b) mode 3 in the simple lattice with 4 bunches. The points are sampled for 1 turn at every 10 turns.

(Hock, Wolski, Phys. Rev. ST Accel. Beams 10, 084401 (2007))

OCS6: Comparison of Effects on Growth Rates

HOM, uneven fill can also produce mode coupling, but their effects are smaller than that of beta function variation in ILC damping ring

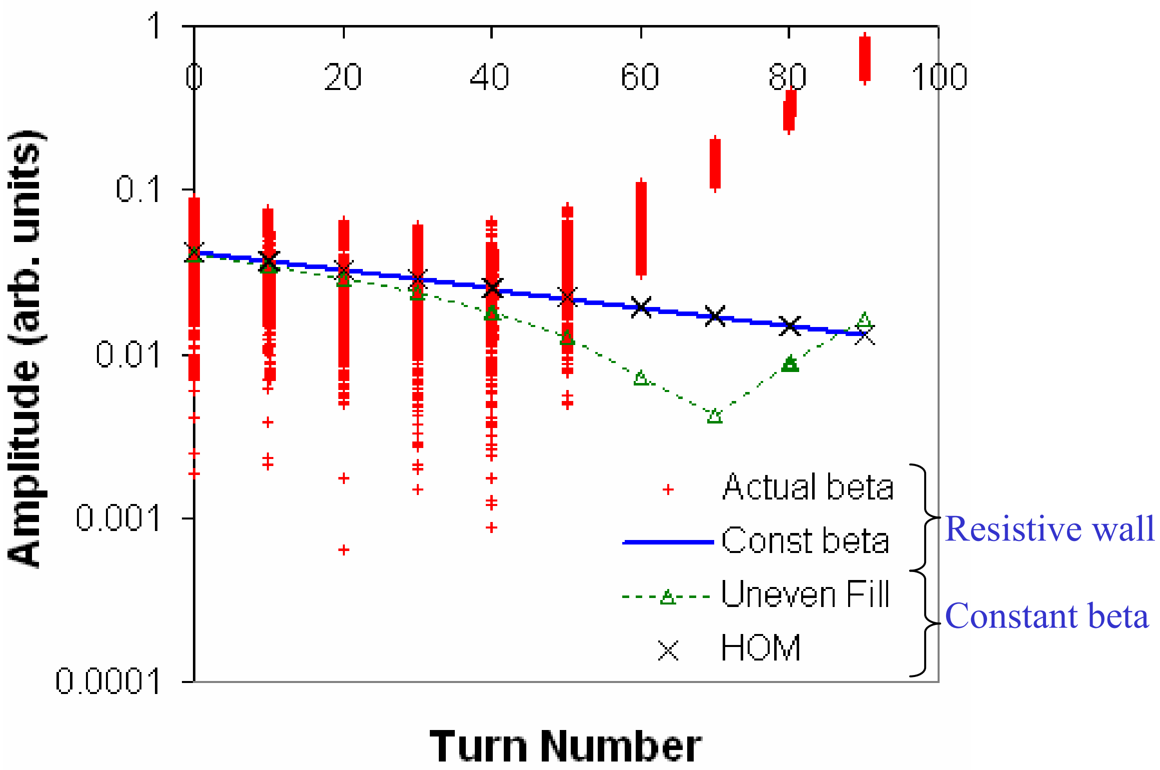


Figure 4. (Color) Simulation results for mode 100 using the actual, varying beta function of the ILC damping ring. The graph shows the amplitude of the normalized action constructed from the data.

Discrepancy due to varying beta

Assuming constant beta function, equations of motion can be decoupled after transforming to Fourier modes:

$$\ddot{\tilde{x}}_\mu(t) + \omega_\beta{}^2 \tilde{x}_\mu(t) = \sum_{n=1} b_n e^{i\frac{2\pi n\mu}{M}} \tilde{x}_\mu(t - n\tau)$$

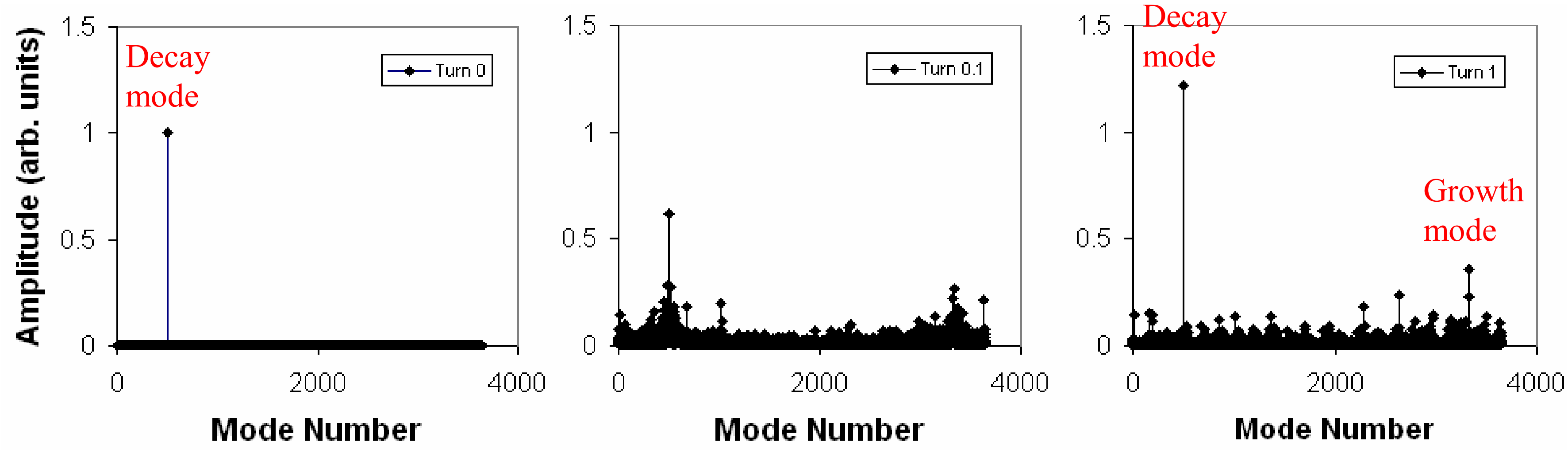
If beta function varies, equations cannot be decoupled. Attempting to do so results in mode coupling term:

$$\ddot{\tilde{x}}_\mu(t) + \overline{K}\tilde{x}_\mu(t) = \sum_{n=1} b_n e^{i\frac{2\pi n\mu}{M}} \tilde{x}_\mu(t - n\tau) - \frac{1}{M}\sum_{\mu'=0}^{M-1} \tilde{k}_{\mu-\mu'}(t)\tilde{x}_{\mu'}(t)$$

Since Fourier modes are no longer eigenmodes, mode mixing can take place.

To illustrate …

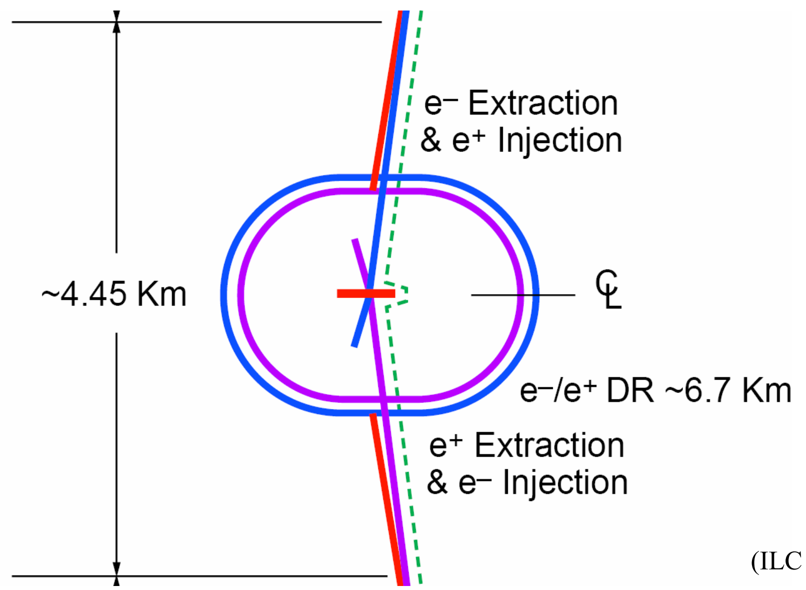
Decay modes grow because Fourier modes are no longer eigenmodes.



NB. All modes shown here are Fourier modes

This also means that the analytic growth rate is no longer valid.

Ongoing Work – Transient Effects



(ILC RDR Vol. 3, 2007, fig. 1.3-1)

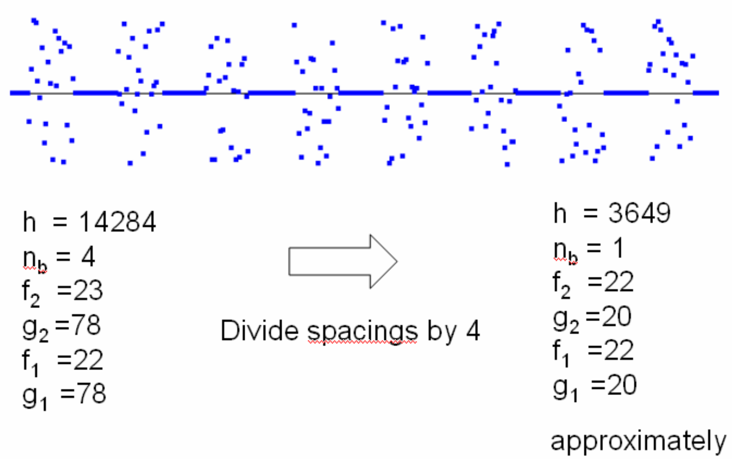
- injection transients → emittance growth
- finite wall wake function
- NEG coating
- feedback system requirements
- wake sum convergence problem
- groove chamber surface

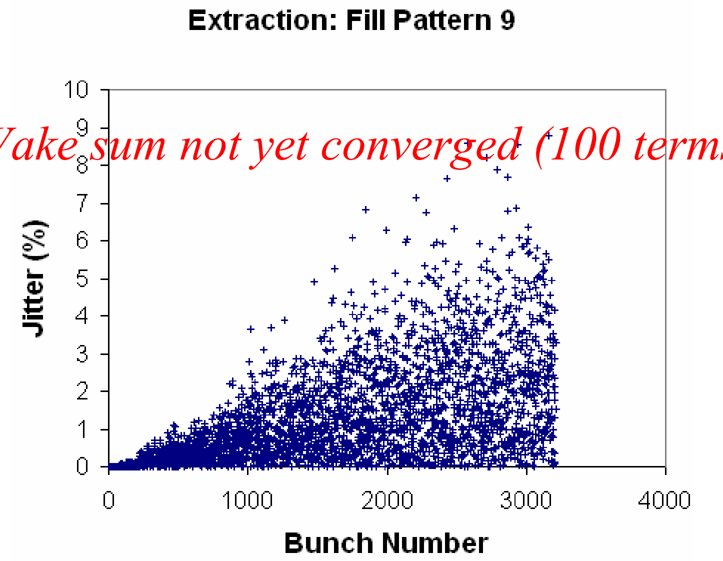
→ OCS8

Injection Transients → Emittance Growth

Continue studies to quantify the effects of injection transients.

Simplify to reduce compute time





Wake sum not yet converged (100 terms)

$$Jitter = \frac{y}{\sigma_y} = \sqrt{\frac{2J}{\varepsilon_y}}$$

$$\varepsilon_y = 2 \text{ pm}$$

Feedback System

Provide information required for the specification and development of the fast feedback systems;

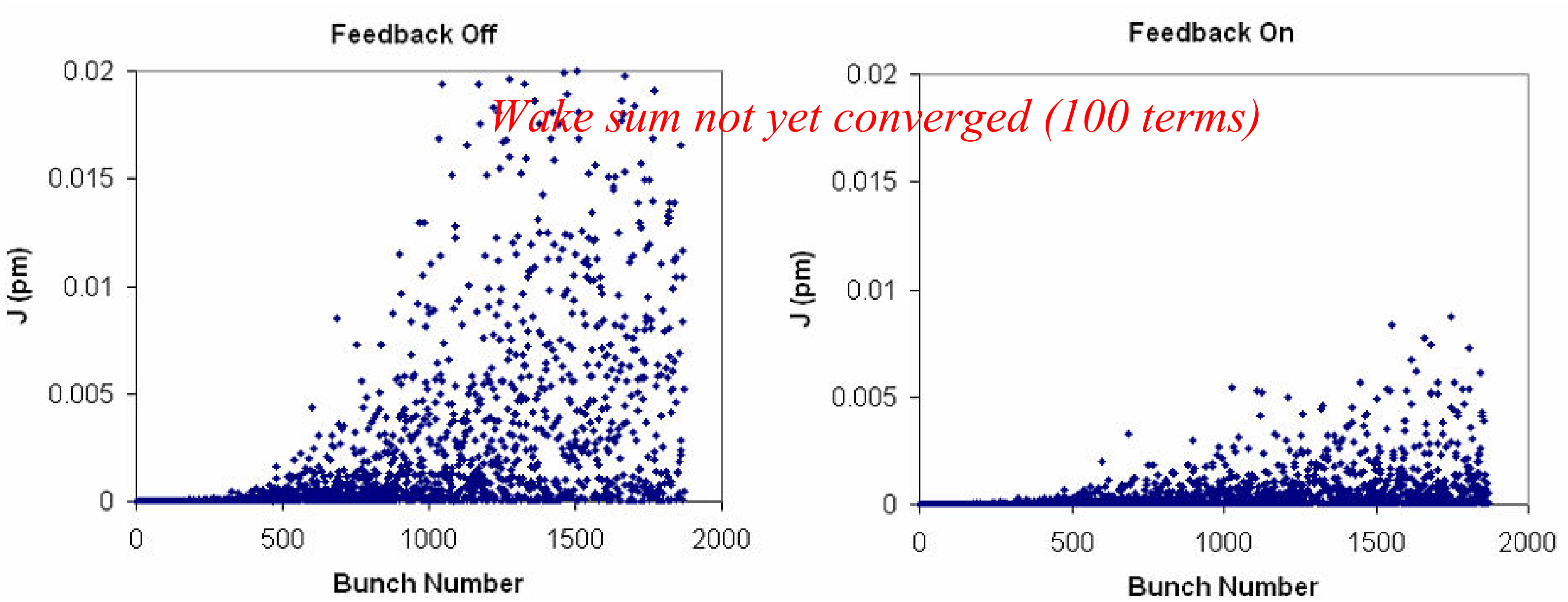
- John Fox (SLAC) is the proposed work package manager for the bunch-by-bunch feedback systems.

Required gain inversely related to growth time, τ

$$\frac{d\tilde{V}}{dy} \geq \frac{2}{\sqrt{\beta_1\beta_2}} \frac{E}{e} \frac{T_0}{\tau}$$

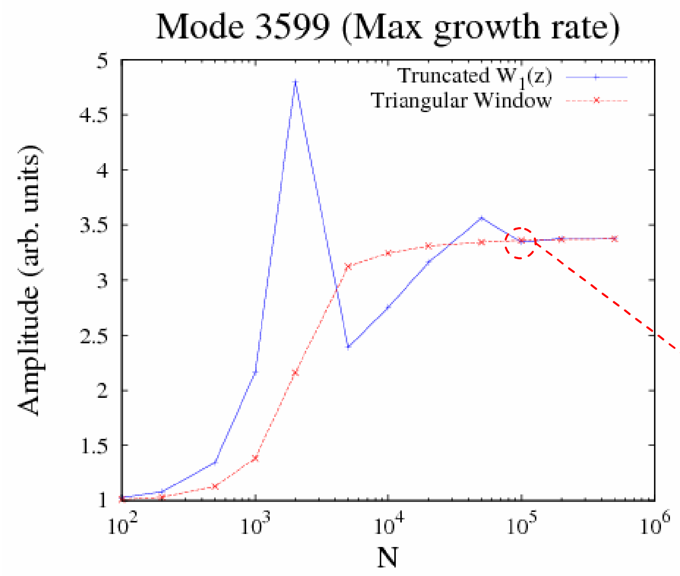
(Wolski 2004)

Performance limited by maximum achievable V

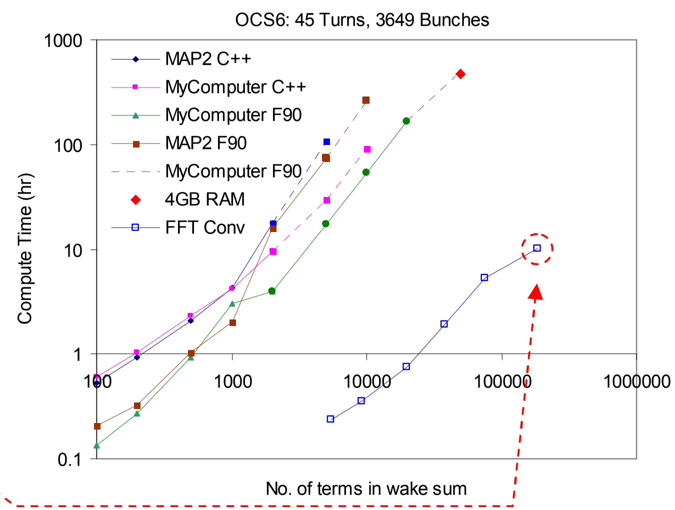


Wake Sum using FFT Convolution

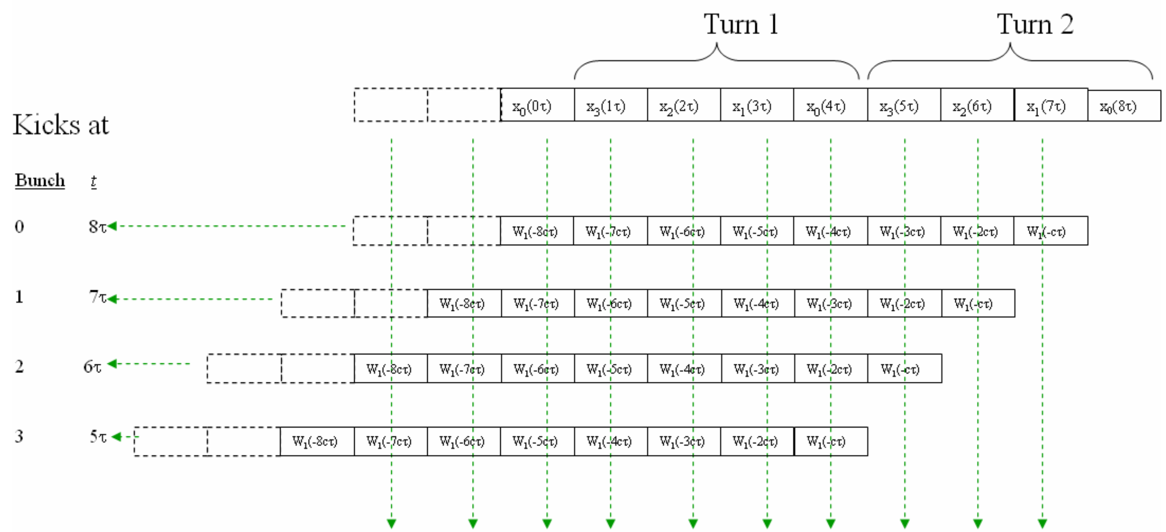
No. of terms needed to converge



Computation time needed



Addition of wakefield terms can be arranged into discrete convolution (Koschik 2004)



Finite Wall Wake Function

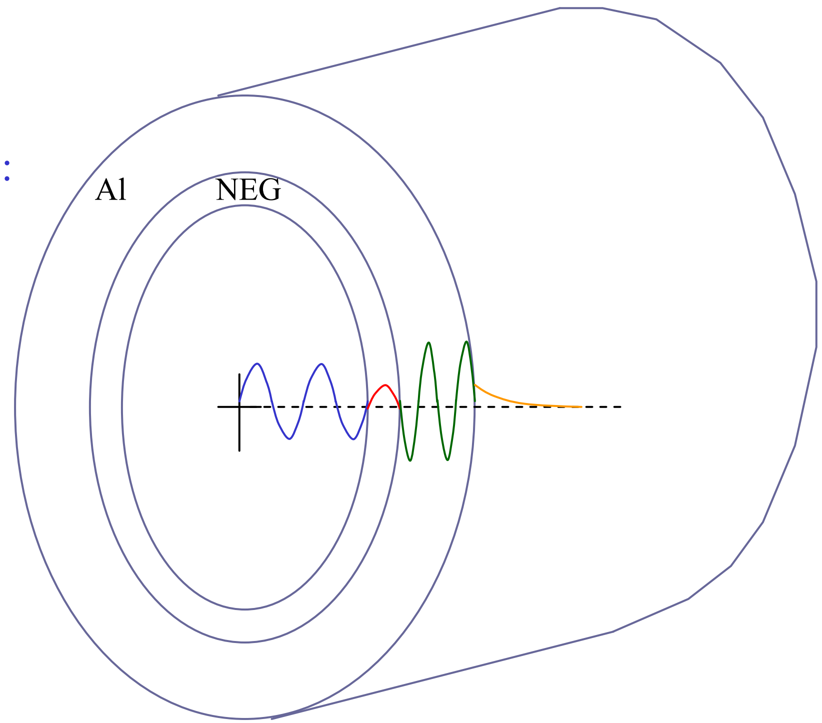
More sophisticated wake function models will be considered to improve accuracy of simulation and to include multilayer walls, e.g. NEG coating

Gluckstern (2000), Zotter (2005), Al-Khateeb (2007):

- Matching of boundary conditions at interfaces gives linear system of equations

- Badly conditioned because coefficients span up to 12 orders of magnitude

- Analytic solution necessary, e.g. Mathematica



NEG Coating – Multi-layer Wake Function

Estimate the impact of NEG coating

- Some work has already been done for other machines: e.g. R. Nagaoka, "Study of resistive-wall effects on Soleil," Proceedings of EPAC'04, Lucerne, Switzerland (2004).

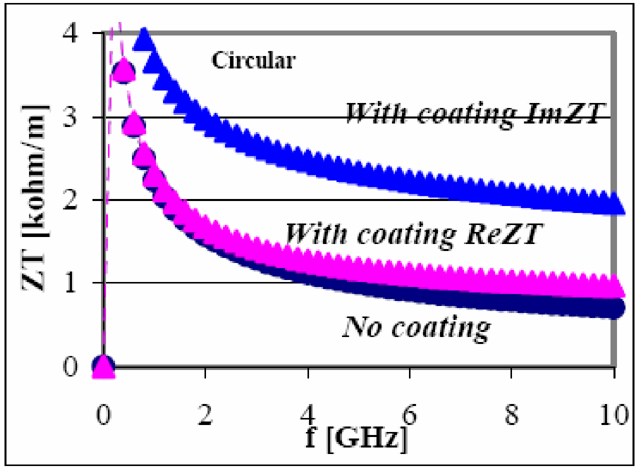


Figure 3: Transverse impedance with and without NEG coating.

Conclusion

1. Analytic result of growth time from resistive wall wakefield is 36.7 turns. Feedback damping time achievable is smaller but of similar order of magnitude.

2. Variation of beta function in real lattice reduces growth time to 30 turns. This could approach the limit of feedback system performance.

3. Wakefield of injected bunches induces vertical jitter in extracted bunches that may exceed 10% for 2 pm emittance.

4. Convergence problem in wakefield sum is solved using FFT convolution technique and will provide more accurate answers on transient effects.

5. More realistic effect of feedback control can now be calculated using time domain simulation.