e-Cloud Theoretical Studies

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Outline

- *Photo-Electrons*
- *Dynamics of Electrons in e-Cloud*
- *e-Cloud Spatial Distribution*
- *Wake in the e-Cloud*
- *Experimental Implications*

Photo-Electrons: synchrotron radiation

J.D. Jackson

$$\frac{N_{ph}(E > E_{th})}{N_e} \simeq 49.54 \times E_k[GeV] \int_{E_{th}/E_{cr}}^{\infty} dy \int_{2y}^{\infty} dx\, K_{5/3}(x)$$

$$V_{ph}(E > E_{th}) = \frac{\mathcal{E}_{ph}(c/2\pi\rho)}{eN_e(c/2\pi\rho)} \simeq 2.193 \times 10^5 \frac{E_k^4(eV)}{\rho[m]} \int_{E_{th}/E_{cr}}^{\infty} dy\, y \int_{2y}^{\infty} dx\, K_{5/3}(x)$$

	$E_{th}[eV]$	$E_k = 2[GeV]$	$E_k = 5[GeV]$
N_{ph} / N_e	0	259.4	648.5
	4	177 (68%)	565 (87%)
	300	13.8(5%)	314 (48%)
V_{ph}	0	177[kV]	0.690[MV]
	4	176[kV]	0.690[MV]
	300	6.6[kV] (4%)	0.667[MV] (97%)

Peak of SE (Al)

Work function (Al)

Photo-Electrons Yield

Photo-electrons spectrum is a "convolution" with the photon's spectrum

$$\tilde{N}_{pe}(E) \simeq \int_{E_w}^{\infty} dE' \, \delta_{pe}(E|E') \left[N_{ph} f_{ph}(E') \right]$$

Similar to the photons we are interested on :

$$\frac{\bar{N}_{pe}(E > E_{th})}{N_e} = 49.6 \times E_k[GeV] \int_{E_{th}}^{\infty} dE \int_{E_w}^{\infty} dE' \frac{1}{E_{cr}} \delta_{pe}(E|E') \int_{2E'/E_{cr}}^{\infty} dx \, K_{5/3}(x)$$

$$V_{pe}(E > E_{th}) = 2.193 \times 10^5 \frac{E_k^4(GeV)}{\rho[m]} \int_{E_{th}}^{\infty} dE \frac{E}{E_{cr}} \int_{E_w}^{\infty} dE' \, \delta_{pe}(E|E') \int_{2E'/E_{cr}}^{\infty} dx \, K_{5/3}(x)$$

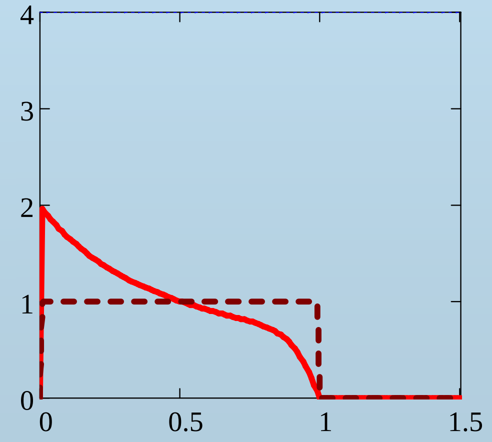
Photo-Electrons – Model for the Yield

1. *According to Grobner et.al [J. Vac. Sci. Technolo. A 7 (2) 1989 p, 223-9 "p.224 (Figure 2)]* *the yield of extracting a photo-electron from an aluminium surface*

$$\delta_{\text{pe}}\left(E \middle| E'\right) \propto Y(E') = \left(\frac{E_0}{E'}\right)^{\nu}$$

Typically: $\begin{cases} E_0 = 2.316[eV] \\ \nu = 0.825 \end{cases}$

2. *The energy of the photo-electron should not exceed the energy of the incident photon and we further assume that the electrons' energy is uniformly distributed between zero and the energy of incoming photon*



$$\delta_{\text{pe}}\left(E \middle| E'\right) \propto \frac{1}{E'}\left[h(E) - h(E - E')\right]$$

Photo-Electrons – Model for the Yield

3. No contribution from photons of energy lower than the work function

$$\delta_{\mathrm{pe}}\left(E|E'\right) \propto h\left(E'-E_{\mathrm{w}}\right)$$

4. Not all the photons are absorbed

Reflection coefficient $\rho\left(E'\right)$

$$\delta_{\mathrm{pe}}\left(E|E'\right) \propto \left[1-\left|\rho_{\mathrm{eff}}\left(E'\right)\right|^2\right]$$

"Handbook of X-rays", edited by E.F. Kaelble, McGraw-Hill, New York, 1967, p. 48-2

$$\left|\rho_{\mathrm{eff}}\left(E\right)\right|^2 \simeq \frac{1.881}{E}\exp\left(-1.2486\times10^{-4}E^2\right)$$

Virtually all photons with energy higher than 100eV are absorbed at the first impact!!

$$\delta_{\mathrm{pe}}\left(E|E'\right) \simeq Y\left(E'\right)\frac{1}{E'}\left[h\left(E\right)-h\left(E-E'\right)\right]h\left(E'-E_{\mathrm{w}}\right)\left[1-\left|\rho_{\mathrm{eff}}\left(E'\right)\right|^2\right]$$

Photo-Electrons –Preliminary Conclusions

Comparable

	$E_{th}[eV]$	$E_k = 2[GeV]$		$E_k = 5[GeV]$	
		$\rho_{eff}(E) = 0$	$\rho_{eff}(E) \neq 0$	$\rho_{eff}(E) = 0$	$\rho_{eff}(E) \neq 0$
N_{pe} / N_e	0	25.8	20.9	32.9	27.6
	300	0.047(0.18%)	0.047(0.22%)	1.1(3.3%)	1.1(4%)
V_{pe}	0	354.4[V]	332.9[V]	1.64[kV]	1.61[kV]
	300	19.8[V](5.6%)	19.8[V](6%)	0.97[kV](59%)	0.97[kV](60%)
$\bar{\delta}_{pe}$	0	0.1(0)	0.8(-1)	0.5(-1)	0.4(-1)
	300	1.8(-4)	1.8(-4)	1.7(-4)	1.7(-4)

One order of magnitude difference

Reflections (Al) may reduce the number of photo-electrons by about 20%.

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Dynamics of Electrons in e-Cloud

$$\frac{\partial \tilde{N}_{\mathrm{ec}}(t,E)}{\partial t} = \frac{1}{\tau(E)}\left[-\tilde{N}_{\mathrm{ec}}(t,E) + \tilde{N}_{\mathrm{se}}(t,E)\right] + \tilde{N}_{\mathrm{pe}}(E)\sum_{n=1}^{N_b}\delta(t-T_n)$$

Life-time

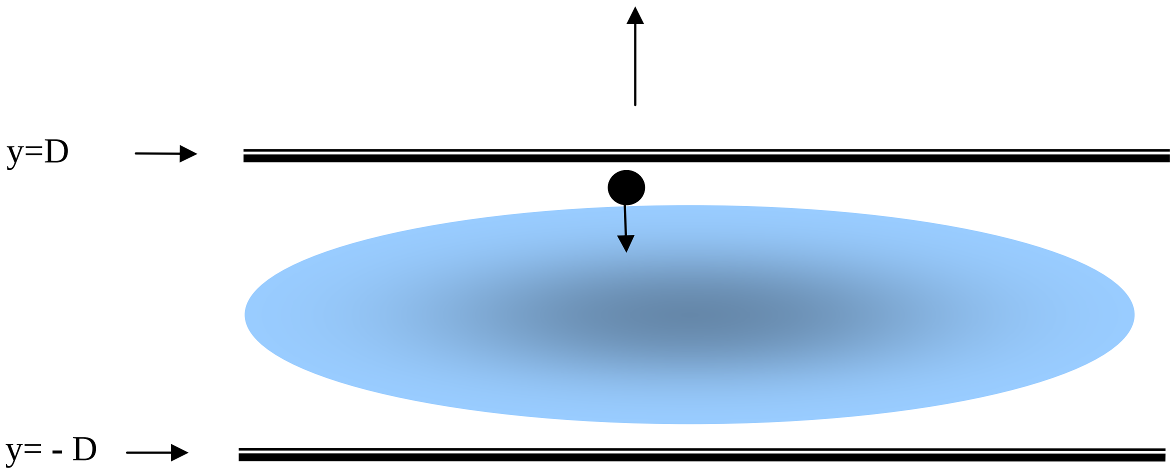
Spectrum of SE

Spectrum of PE

Bunch separation is much longer than the duration of single bunch

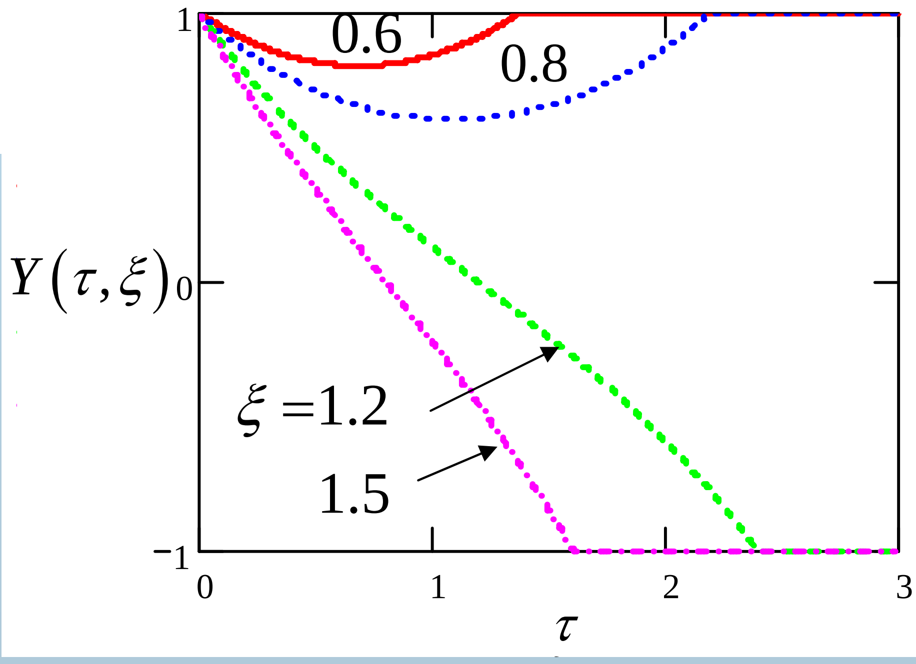
..or Ionization electrons

Dynamics of Electrons in e-Cloud $\tau(E) = ??$



$$\frac{d^2y}{dt^2} = -\frac{e}{m}E_y = \frac{e^2 n_0}{m\varepsilon_0}y = \omega_p^2 y$$

$$Y\left(\tau \equiv \omega_p t, \xi \equiv \frac{V_0}{\omega_p D}\right) \equiv \frac{y(t)}{D} = \cosh(\tau) - \xi \sinh(\tau)$$



Dynamics of Electrons in e-Cloud

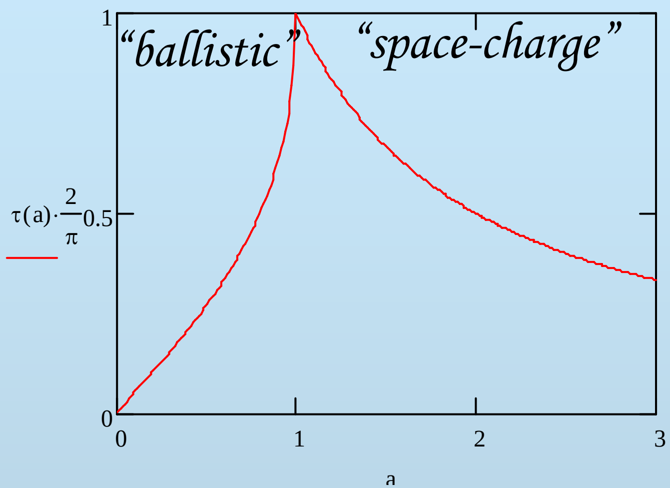
$$\tau(E) \simeq \frac{\pi}{\omega_p}\begin{cases} \dfrac{\sqrt{E_0/E}}{2-(E_0/E)^2} & E > E_0 \\ \sqrt{E/E_0} & E < E_0 \end{cases}$$

$$E_0 = \frac{1}{2} m\omega_p^2 a_y^2$$

$$E = E_0 = 100[eV],\ \ D_y = 2.5[cm]$$

$$n_{\max}[m^{-3}] = 1.1\times 10^{12}\frac{E[eV]}{a_y^2[cm]} = 1.76\times 10^{13}$$

$$\overline{\tau}^{(\max)}[n\sec] = 52.93\frac{a_y[cm]}{\sqrt{E[eV]}} = 13.2$$



Note that the life-time of the electron in the cloud may be comparable to the bunch separation !!!

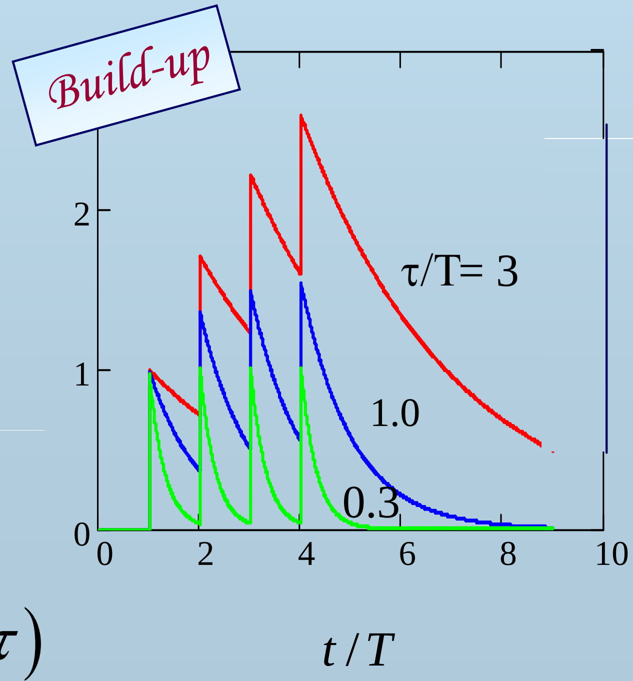
Dynamics of the e-Cloud

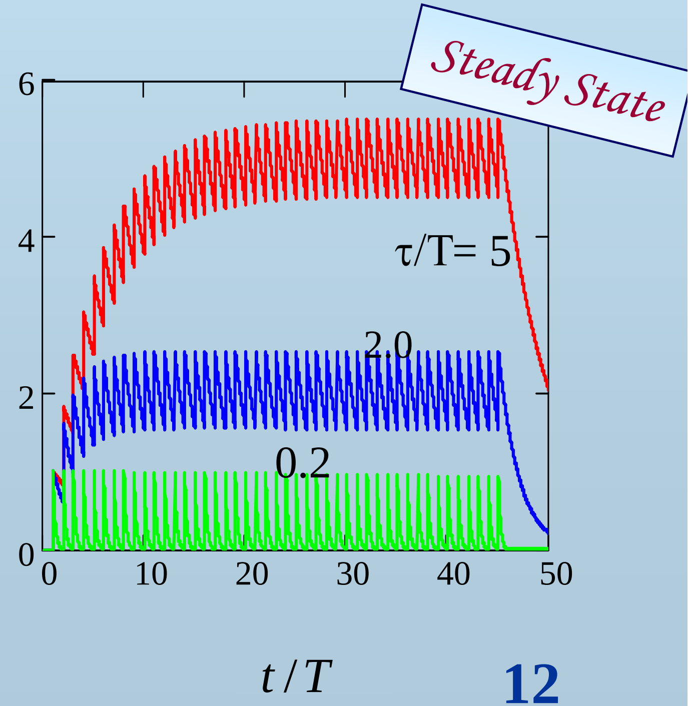
Let us consider the dynamics w/o SE

$$\frac{\partial \tilde{N}_{\text{ec}}(t,E)}{\partial t} = \frac{1}{\tau(E)}\left[-\tilde{N}_{\text{ec}}(t,E) + \tilde{N}_{\text{se}}(t,E)\right] + \tilde{N}_{\text{pe}}(E)\sum_{n=1}^{N_b}\delta(t-T_n)$$

$$\tilde{N}_{\text{ec}}(t,E) = \tilde{N}_{\text{pe}}(E)\sum_{n=1}^{N_b} h(t-T_n)\exp\left[-\frac{t-T_n}{\tau(E,\bar{n}_{\text{ec}})}\right]$$

Both build-up and steady-state depend on the ratio T/τ





$$y_{n+1} = (y_n + \Delta)\exp(-T/\tau)$$

Dynamics of Electrons in e-Cloud

In practice, we need to integrate over the entire spectrum

$$\bar{n}_{\text{ec}}(t) \equiv \frac{1}{(D_x D_y)(2\pi\rho)} \int_0^\infty dE\, \tilde{N}_{\text{ec}}(t,E)$$

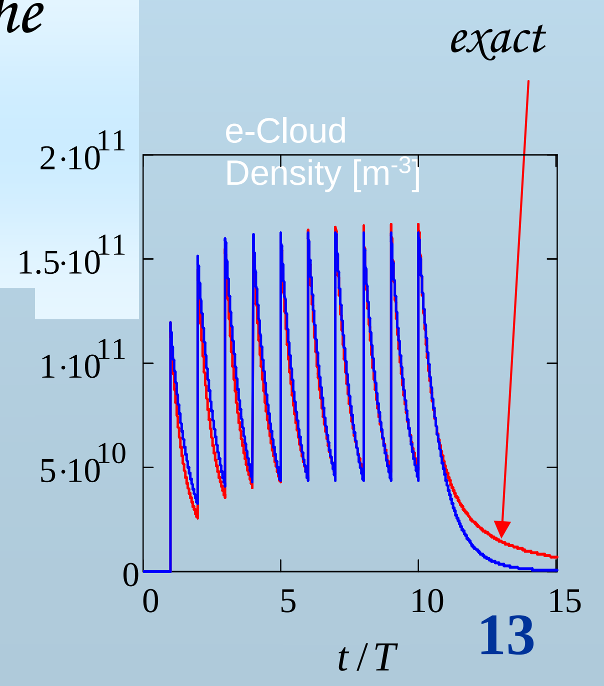
$$= \frac{\bar{N}_{\text{pe}}}{(D_x D_y)(2\pi\rho)} \sum_{n=1}^{N_b} h(t-T_n) \int_0^\infty dE\, f_{\text{pe}}(E) \exp\left[-\frac{t-T_n}{\tau(E,\bar{n}_{\text{ec}})}\right]$$

For the parameters involved, we found that defining the average life-time

$$\bar{\tau}(\bar{n}_{\text{ec}}) \equiv \int_0^\infty dE\, f_{\text{pe}}(E)\, \tau(E,\bar{n}_{\text{ec}})$$

is convenient since

$$\bar{n}_{\text{ec}}(t) = \frac{\bar{N}_{\text{pe}}}{(D_x D_y)(2\pi\rho)} \sum_{n=1}^{N_b} h(t-T_n) \exp\left[-\frac{t-T_n}{\bar{\tau}(\bar{n}_{\text{ec}})}\right]$$



No significant difference between the two

Dynamics of Electrons in e-Cloud

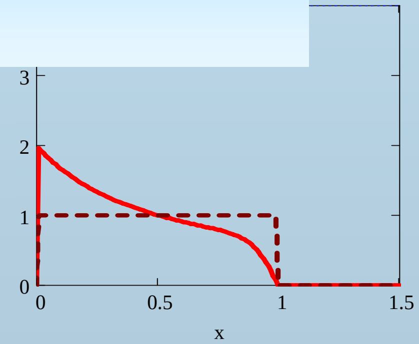
And now with secondary emission included:

$$\tilde{N}_{se}(t,E) = \int_0^\infty dE' \delta_{se}(E|E') \tilde{N}_{ec}(t,E')$$

Assumption: *Because of the vertical magnetic field (bend or wiggler) the electrons gyrate along the magnetic field line consequently, both incoming and outgoing electrons are assumed to be moving in the vertical direction only.*

Without significant loss of generality, based on the experimental data, it is reasonable to further assume that the energy of the secondary electrons is uniformly distributed between zero and that of the incident electron

$$\delta_{se}(E|E') \simeq \left[h(E) - h(E-E')\right] \frac{\tilde{\delta}_{se}(E')}{E'}$$



Consequently,

$$\frac{\partial \tilde{N}_{ec}(t,E)}{\partial t} = \frac{-1}{\tau(E)} \left[\tilde{N}_{ec}(t,E) - \int_E^\infty dE' \frac{\tilde{\delta}_{se}(E')}{E'} \tilde{N}_{ec}(t,E') \right] + \tilde{N}_{pe}(E) \sum_{n=1}^{N_b} \delta(t - T_n)$$

Dynamics of Electrons in e-Cloud

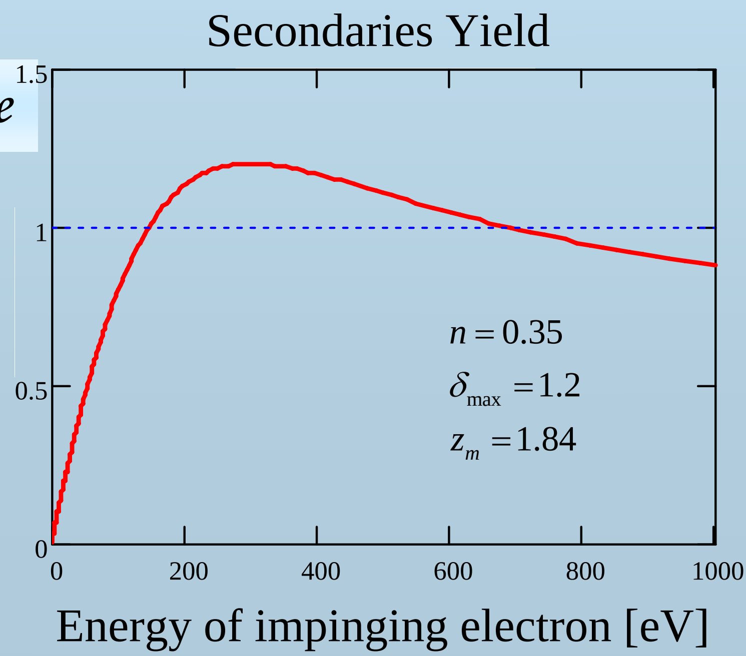
A realistic model for secondary emission [J.R.M Vaughan, "A new Formula for Secondary Emission Yield", IEEE Transactions on Electron Devices Vol 36 (9) p.1963 (1989). Actually a variant of: R.G. Lye and A.J. Dekker, "Theory of Secondary Emission", Physical Review Vol. 107, pp.977-981 (1957).]:

$$\tilde{\delta}_{\text{se}}(E') = \delta_{\max} \frac{1}{g_n(z_{\text{m}})} g_n\left(z_{\text{m}} \frac{E}{E_{\max}} \right)$$

$$g_n(z) = \left[1 - \exp\left(-z^{n+1}\right)\right] z^{-n}$$

Where the typical values of the two parameters are

$$n \simeq 0.35,\ z_m \simeq 1.84$$



Dynamics of Electrons in e-Cloud

Now we can proceed to a solution of

$$\frac{\partial \tilde{N}_{\text{ec}}(t,E)}{\partial t} = \frac{-1}{\tau(E)}\left[\tilde{N}_{\text{ec}}(t,E) - \int_E^\infty dE' \frac{\tilde{\delta}_{\text{se}}(E')}{E'} \tilde{N}_{\text{ec}}(t,E')\right] + \tilde{N}_{\text{pe}}(E)\sum_{n=1}^{N_b}\delta(t-T_n)$$

without SE we found

$$\tilde{N}_{\text{ec}}(t,E) = \tilde{N}_{\text{pe}}(E)\sum_{n=1}^{N_b} h(t-T_n)\exp\left[-\frac{t-T_n}{\tau(E)}\right]$$

with SE the solution is

$$\tilde{N}_{\text{ec}}(t,E) = \tilde{N}_{\text{pe}}(E)\sum_{n=1}^{N_b} h(t-T_n)\exp\left[-\frac{t-T_n}{\tau_{\text{ec}}(E)}\right]$$

wherein

$$\tau_{\text{ec}}(E) = \tau(E) + \frac{\exp[-\psi(E)]}{\tilde{N}_{\text{pe}}(E)}\int_E^\infty dE' \exp[\psi(E)] \frac{\tilde{\delta}_{\text{se}}(E')}{E'} \tilde{N}_{\text{pe}}(E')\tau(E')$$

$$\psi(E) = \int_0^E dE' \frac{\tilde{\delta}_{\text{se}}(E')}{E'}$$

Dynamics of Electrons in e-Cloud

The average density during the train duration is

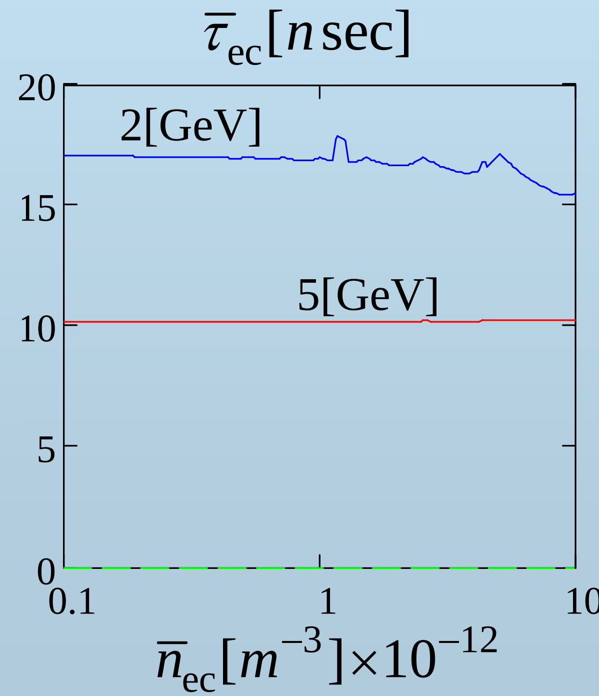
$$\bar{n}_{\rm ec} \equiv \frac{1}{\left(D_x D_y\right)\left(2\pi\rho\right)} \frac{1}{T N_b} \int_0^\infty dt \int_0^\infty dE \tilde{N}_{\rm ec}\left(t, E\right)$$

$$= \frac{\bar{N}_{\rm pe}}{\left(D_x D_y\right)\left(2\pi\rho\right)} \frac{1}{T} \int_0^\infty dE \, f_{\rm pe}\left(E\right) \tau_{\rm ec}\left(E\right)$$

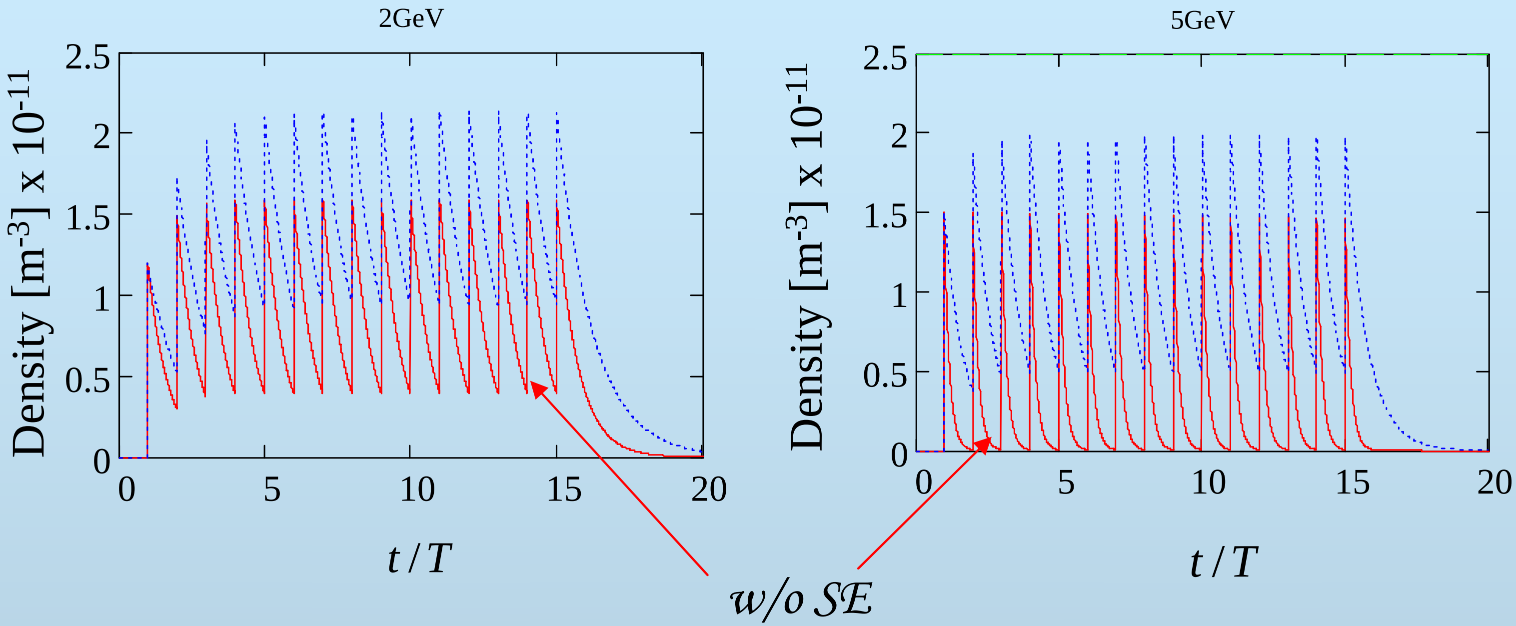
$$\bar{\tau}_{\rm ec} \equiv \int_0^\infty dE \, f_{\rm pe}\left(E\right) \tau_{\rm ec}\left(E\right)$$

- *For a 5GeV train, the average decay is 10nsec in comparison to about 2.5nsec when no SE were present.*
- *At 2GeV the average decay increases from 10 to 17nsec .*
- *In both cases the average decay is virtually independent of the density.*

This result can be attributed to SE being significantly slower than the PE and as a result, it takes them longer time to return to the wall or traverse the gap



Dynamics of Electrons in e-Cloud



Relying on discharge analogy

$$N_{\mathrm{ec}}^{(\mathrm{low})} = \frac{\bar{N}_{\mathrm{pe}}}{\exp\left(T/\bar{\tau}_{\mathrm{ec}}\right) - 1}$$

$$N_{\mathrm{ec}}^{(\mathrm{high})} \simeq \bar{N}_{\mathrm{pe}} + \frac{\bar{N}_{\mathrm{pe}}}{\exp\left(T/\bar{\tau}_{\mathrm{ec}}\right) - 1} = \frac{\bar{N}_{\mathrm{pe}}}{1 - \exp\left(-T/\bar{\tau}_{\mathrm{ec}}\right)}$$

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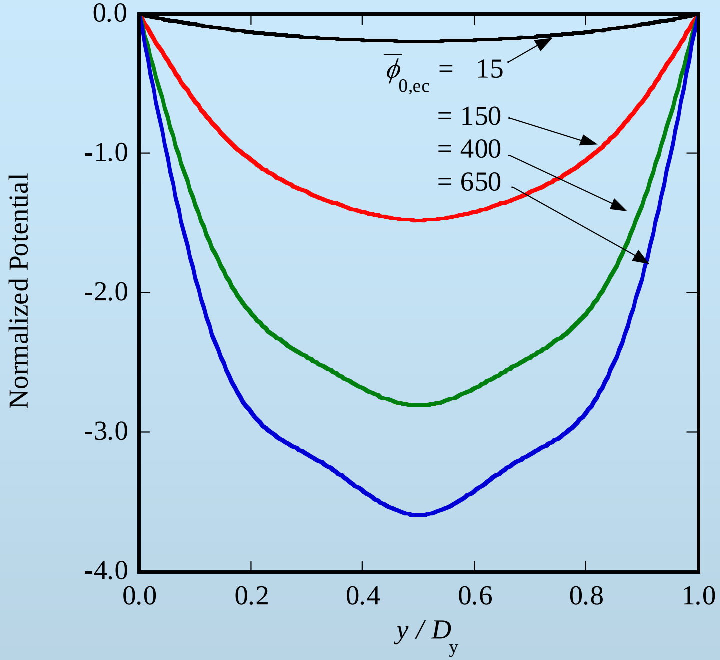
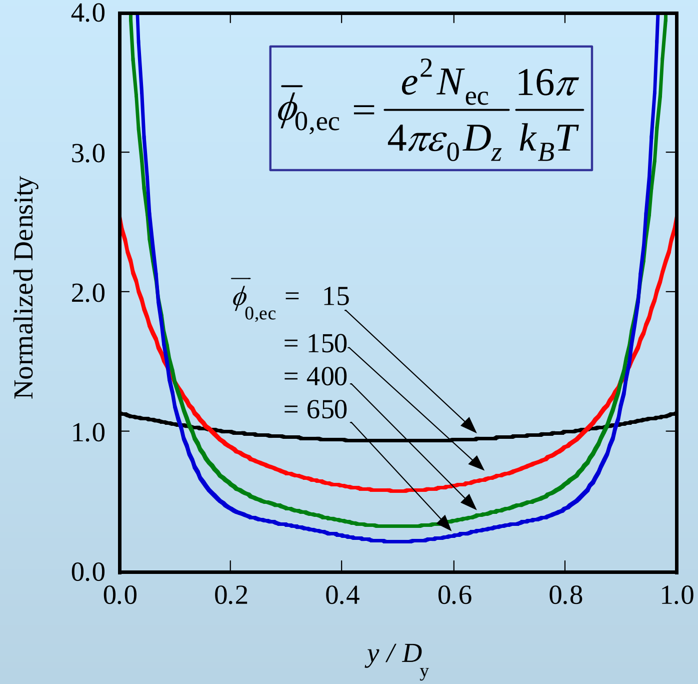
e-Cloud Static Spatial Distribution





Note that in any case, in the center , the potential well may be approximated by the potential of an harmonic oscillator.

$$\bar{\phi}\left(\bar{y}\right) \simeq \bar{\phi}\left(\bar{y}=1/2\right) + \left.\frac{d\bar{\phi}}{d\bar{y}}\right|_{\bar{y}=1/2}\delta\bar{y} + \frac{1}{2}\left.\frac{d^2\bar{\phi}}{d\bar{y}^2}\right|_{\bar{y}=1/2}\delta\bar{y}^2 \qquad \kappa = \left.\frac{d^2\bar{\phi}}{d\bar{y}^2}\right|_{\bar{y}=1/2} = -\sum_{n=1}^{\infty}\left(\pi n\right)^2\bar{\varphi}_n\sin\left(\frac{\pi}{2}n\right)$$

Spring coefficient

e-Cloud Static Spatial Distribution

Implication on the vertical tune. *Let the oscillatory motion (equilibrium) in the absence of the e-cloud be described by*

$$\left(\frac{d^2}{dt^2}+\frac{c^2}{\beta_0^2}\right)\delta y = 0$$

In the presence of the e-cloud

$$\left(\frac{d^2}{dt^2}+\frac{c^2}{\beta_0^2}\right)\delta \overline{y} = \frac{-e}{m\gamma D_y}\left(-\frac{\partial \phi}{\partial y}\right) \simeq \frac{5}{m\gamma D_y^2}\frac{e^2 N_{ec}}{4\pi\varepsilon_0 D_z}\delta \overline{y}$$

Implying

$$\frac{1}{\beta^2} = \frac{1}{\beta_0^2} - \frac{5N_{ec}r_e}{\gamma D_y^2 D_z} = \frac{1}{\beta_0^2} - 5n_{ec}\frac{r_e}{\gamma}\frac{D_x}{D_y}$$

$$\frac{\Delta\nu}{\nu_0} = \sqrt{1-5\frac{n_{ec}r_e\beta_0^2}{\gamma}\frac{D_x}{D_y}} - 1 \simeq -\frac{5}{2}\frac{n_{ec}r_e\beta_0^2}{\gamma}\frac{D_x}{D_y} = -Vn_{ec}$$

During the passage of the bunch, temporarily , the distribution changes.
Electrons repel the cloud and positron attract it making the potential shallower.

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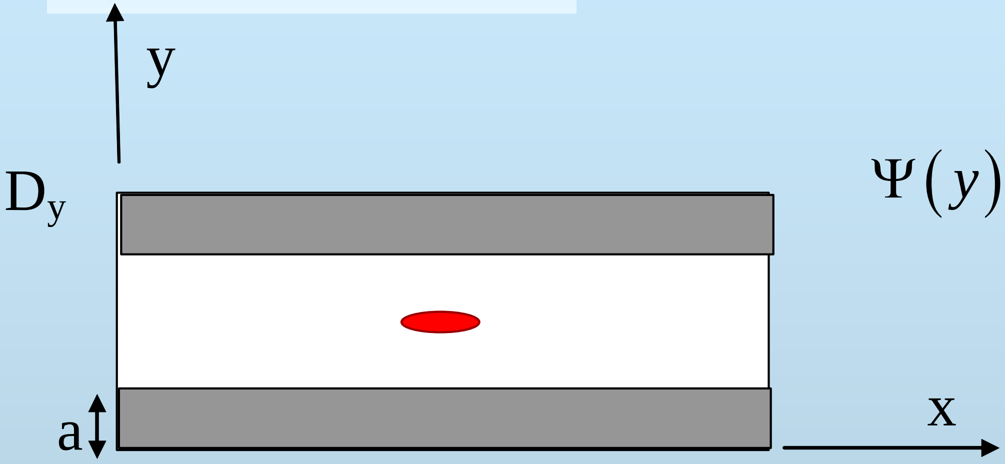
Wakes in the e-Cloud

Eigen-frequencies



$$E_y(x,y,z) = \Psi(y)\sin\left(\pi n_x \frac{x}{D_x}\right)\exp\left(-j\frac{\omega}{V}z\right)$$

$$\Psi(y) = \begin{cases} A\cosh(\Gamma y) & 0 < y < a \\ B\sinh\left[\frac{\pi n_x}{D_x}\left(y - \frac{D_y}{2}\right)\right] & a < y < D_y/2 \end{cases}$$

$$\Gamma^2 = \left(\frac{\pi^2 n_x^2}{D_x^2} + \frac{\omega_p^2}{c^2}\right)\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1}, \quad \Gamma a = j\xi$$

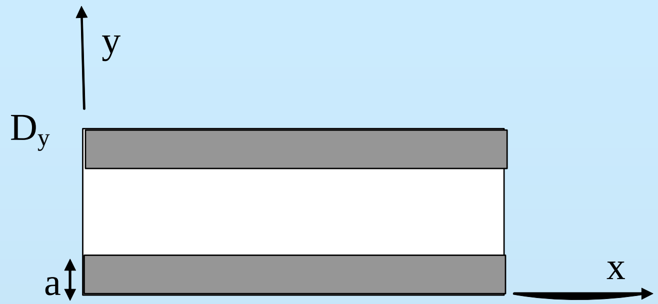
$$\boxed{\tan(\xi) = \frac{\xi_0}{\xi}}$$

$$\kappa_{n_x}^2 \equiv \left(\frac{\pi n_x a}{D_x}\right)^2 + \left(\frac{\omega_p}{c}a\right)^2$$

$$\omega_i^2 = \omega_p^2 \frac{\xi_i^2}{\xi_i^2 + \kappa_{n_x}^2}$$

$$\xi_0 \equiv \left(\frac{\pi n_x a}{D_x}\right)\coth\left(\frac{\pi n_x a}{D_x}\left(\frac{D_y}{2a} - 1\right)\right)$$

Wakes in the e-Cloud



Generated Power

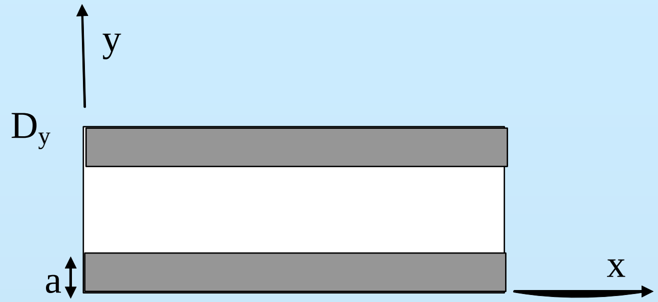
$$P = \int dx \int dy \int dz J_z E_z$$

$$\bar{P} = \frac{-P}{\eta_0 \left(\frac{q_e c}{D_x}\right)^2} = 2\left(\frac{a}{D_x}\right)\left(\frac{\omega_p}{c} D_x\right)^2 \sum_{n_x=0}^{\infty} \frac{\sin^2\left(\frac{\pi}{2} n_x\right)}{\sinh^2\left(\psi_{n_x}\right)} \left(\pi n_x \frac{a}{D_x}\right)^2$$

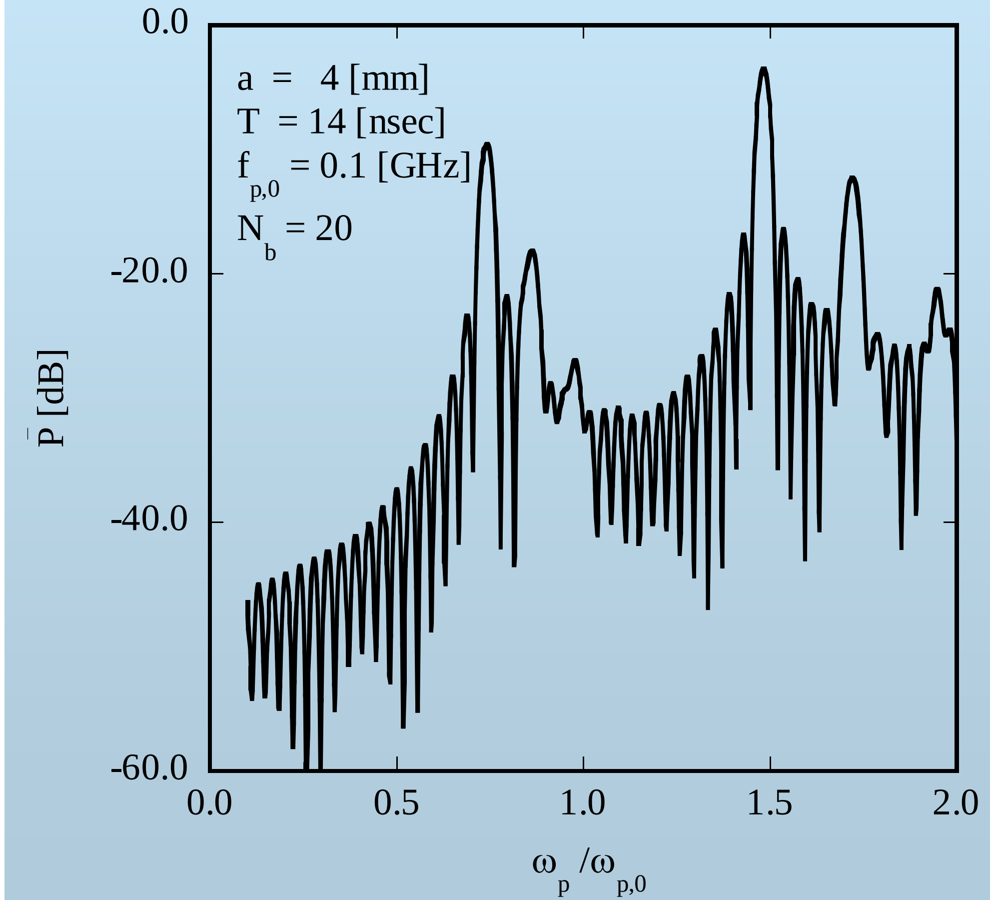
$$\times \sum_i \frac{\xi_i^2 \left[\xi_i^2 + \left(\pi n_x \frac{a}{D_x}\right)^2\right]^{-1}}{\left[\xi_i^2 + \xi_0 \left(1 + \xi_0\right)\right]\left(\xi_i^2 + \kappa_{n_x}^2\right)} \left[\frac{N_b^2}{2} \frac{\operatorname{sinc}^2\left(\frac{1}{2}\omega_i T N_b\right)}{\operatorname{sinc}^2\left(\frac{1}{2}\omega_i T\right)}\right]$$

Eigen-frequencies

Wakes in the e-Cloud



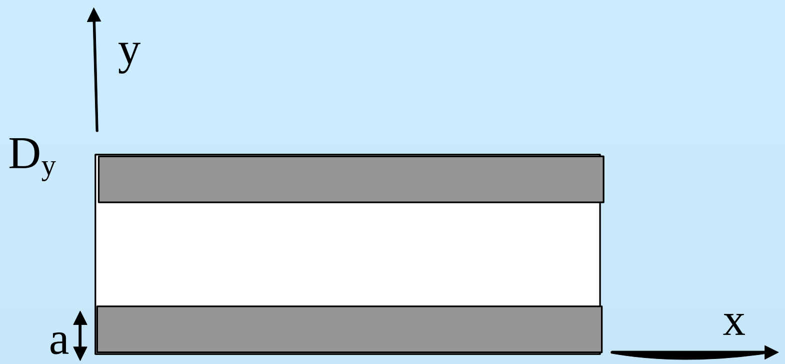
Generated Power



- *Cloud-density controls emitted power*
- *Longer train, sharper peaks*
- *At peak value (infinite life-time)*

$$P \sim -\eta_0 \left(e N_e c / D_x \right)^2 N_b^2 / 2$$

Wakes in the e-Cloud



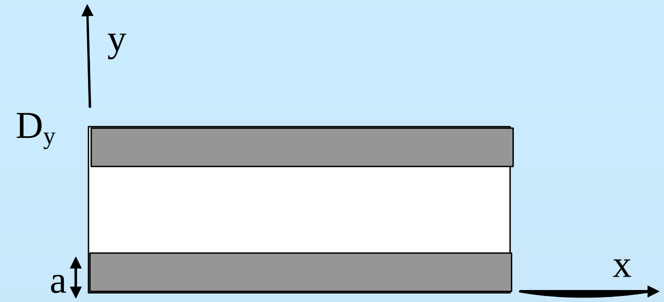
Transverse Kick: "spring coefficient"

$$\bar{K} = \frac{f_y(y) D_x c}{\eta_0 \left(\frac{q_e c}{D_x}\right)^2 \left[g\left(y, \Delta_y\right) \left(y - \frac{D_y}{2} \right) \right]}$$

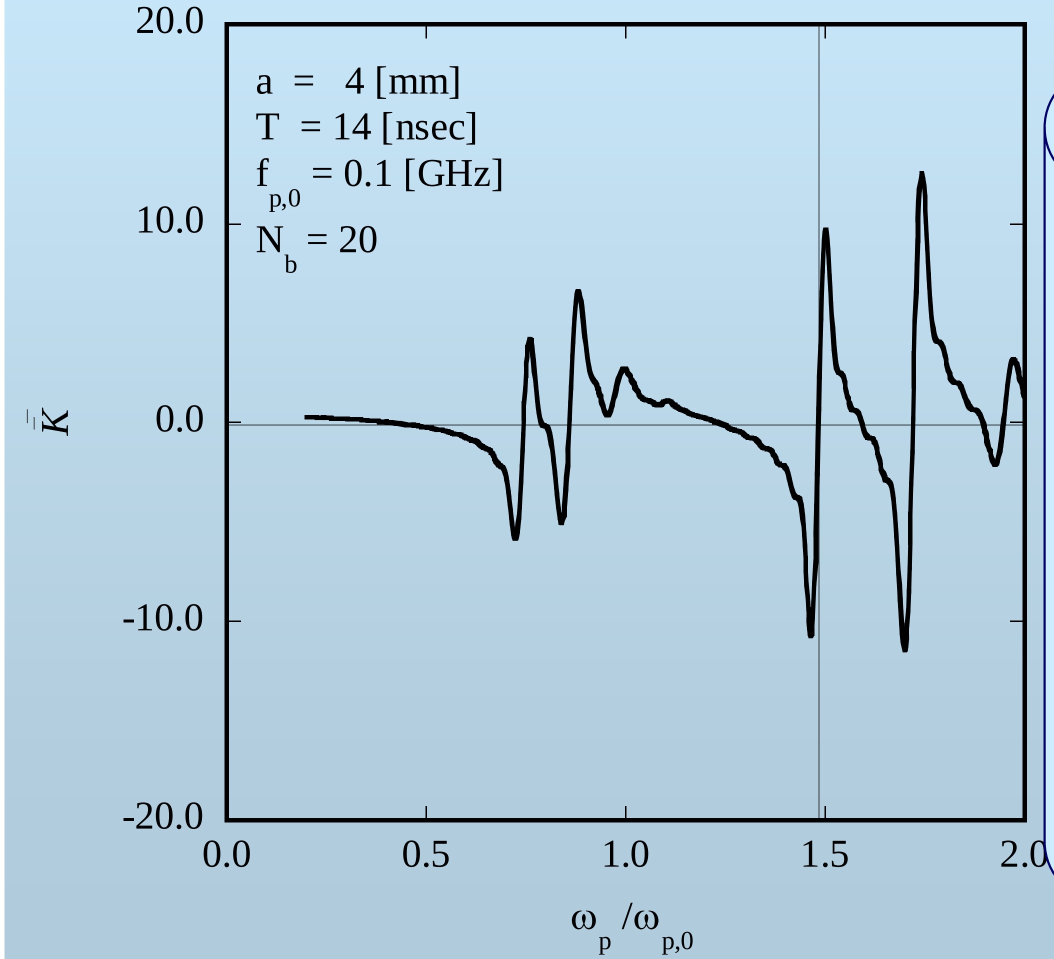
$$= 2\left(\frac{\omega_p}{c} D_x\right)^2 \frac{Tc}{a} \sum_{n_x=0}^{\infty} \left(\pi n_x \frac{a}{D_x}\right)^4 \frac{\sin^2\left(\pi n_x \frac{1}{2}\right)}{\sinh^2 \psi_{n_x}}$$

$$\times \sum_i \frac{\xi_i^2 \left[\xi_i^2 + \left(\frac{\pi n_x a}{D_x}\right)^2 \right]^{-1}}{\left[\xi_i^2 + \chi_{n_x}\left(1 + \chi_{n_x}\right)\right]\left(\xi_i^2 + \kappa_{n_x}^2\right)} N_b \frac{\mathrm{sinc}\left(\omega_i T\right) - \mathrm{sinc}\left(\omega_i T N_b\right)}{\left(\omega_i T\right)^2 \mathrm{sinc}^2\left(\frac{1}{2}\omega_i T\right)}$$

Wakes in the e-Cloud



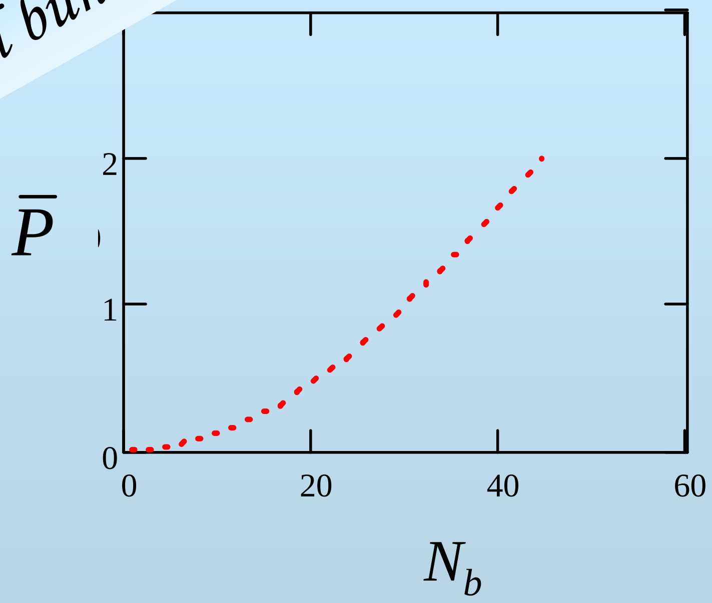
Transverse Kick: “spring coefficient”

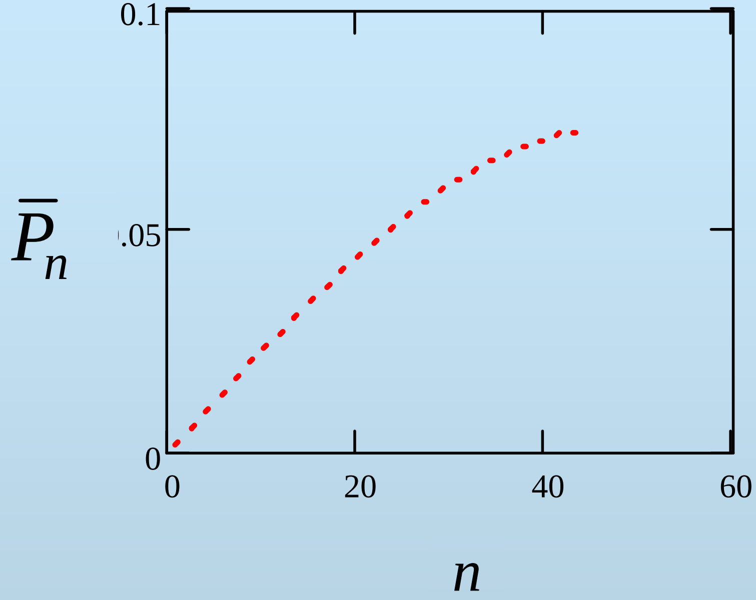


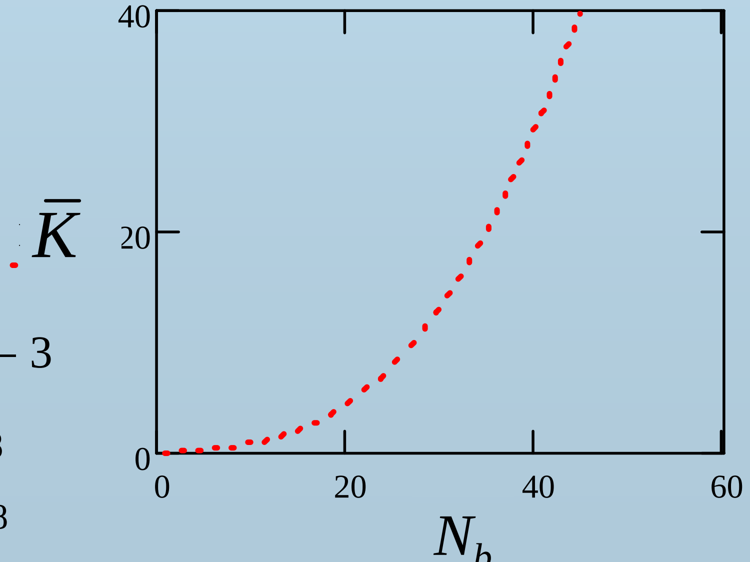
The spring coefficient as a function of the cloud density, may be positive in which case the vertical dynamics is *unstable* *or it can become stable for densities that K<0 .*
This behavior is affected by both the cloud density and the cloud thickness

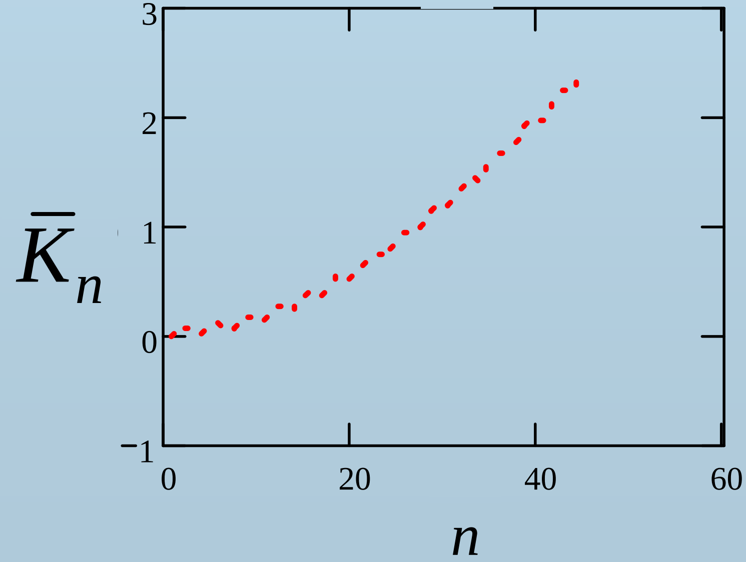
Wakes in the e-Cloud

Effect on the individual bunch









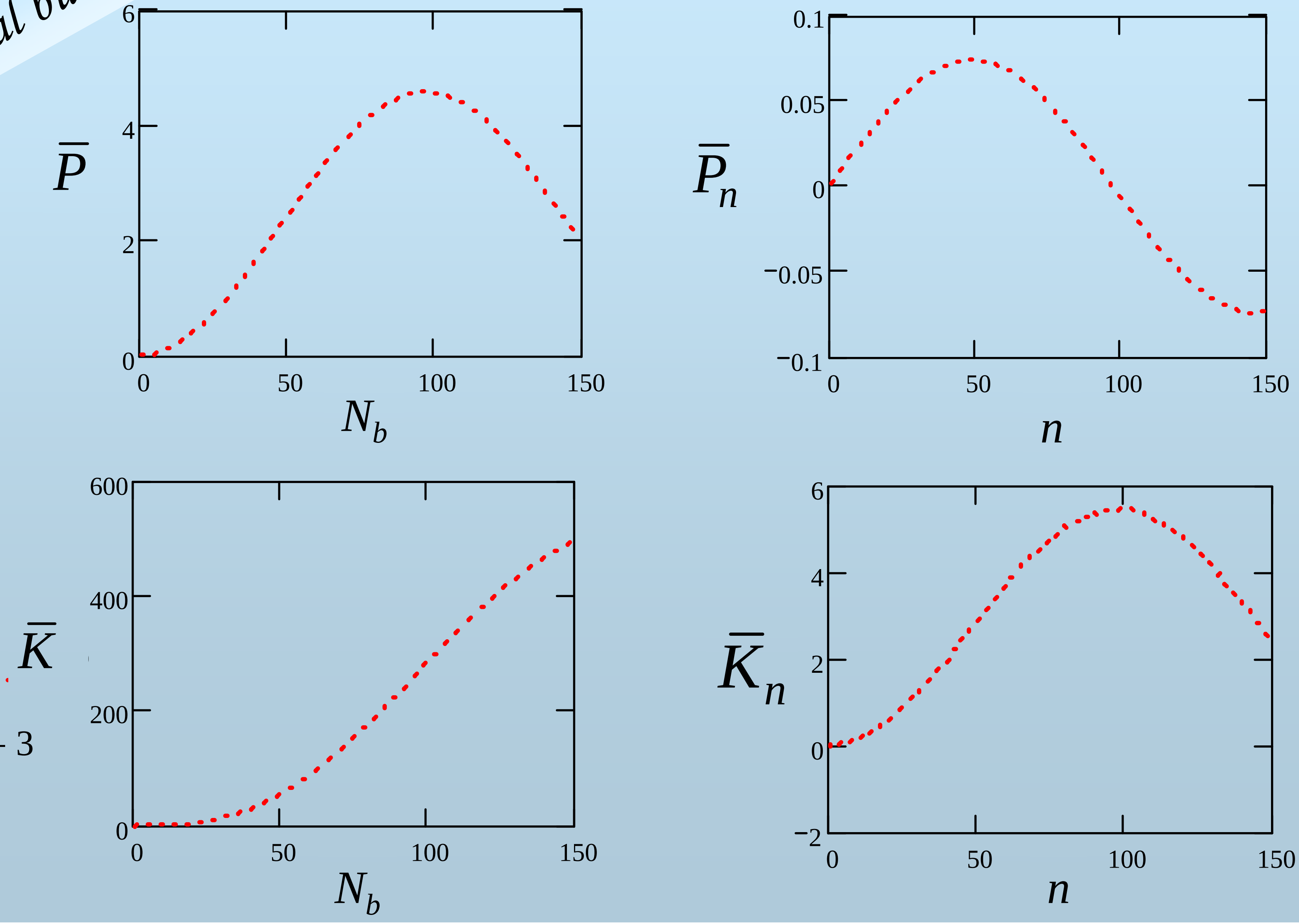
$a = 4.3 \times 10^{-3}$

$T = 1.4 \times 10^{-8}$

$f_p = 1.483 \times 10^{8}$

Wakes in the e-Cloud

Effect on the individual bunch

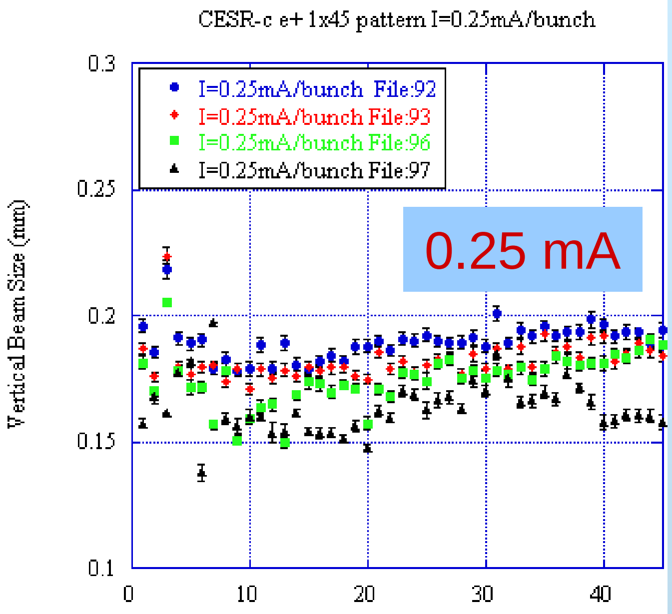


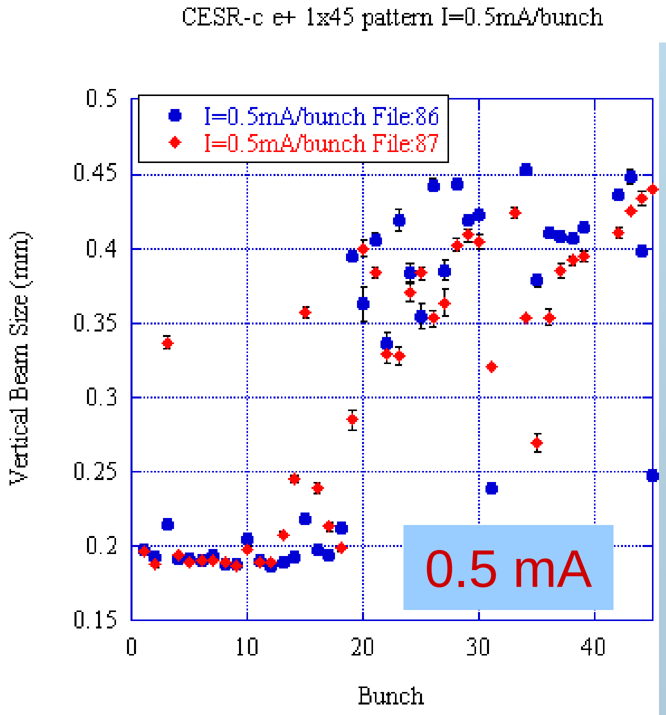
$a = 4.3 \times 10^{-3}$

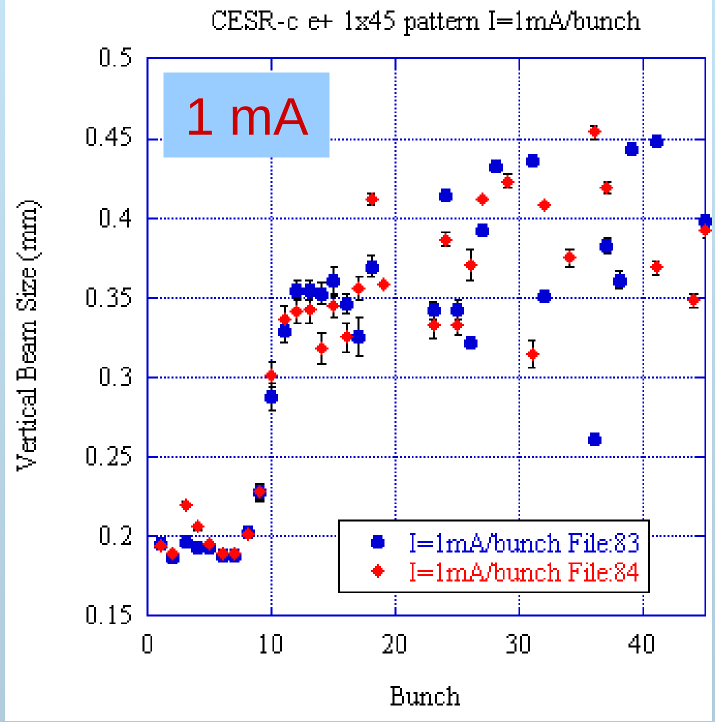
$T = 1.4 \times 10^{-8}$

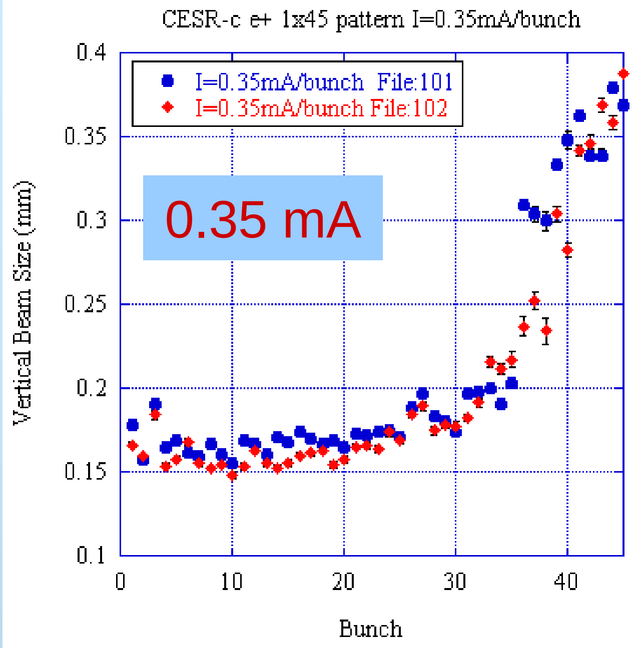
$f_p = 1.483 \times 10^{8}$

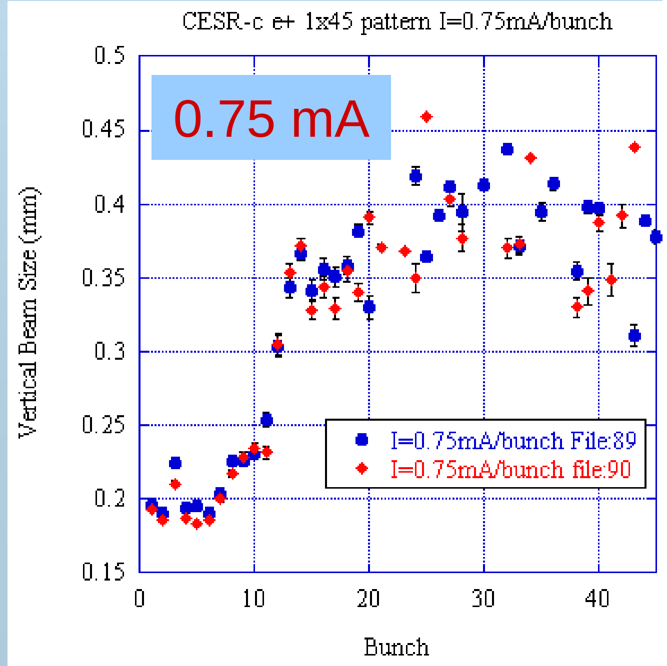
Vertical Beam-size for 1x45 Pattern











Outline

- Photo-Electrons
- Dynamics of Electrons in e-Cloud
- e-Cloud Spatial Distribution
- Wake in the e-Cloud
- Experimental Implications

Experimental Implications

- Radiation generated in the wiggler is the main source of PE.
- Life-time of the electrons in the cloud plays crucial role.
- SE extend the life-time of the cloud.
- Relative tune shift proportional to the average cloud density.
- Potential distribution: main source of asymmetry between e^- & e^+.
- "Vacuum Cleaner" may be efficient in particular for high N_{ec}. Repels ions from the surface.
- Space-charge waves in the cloud generate microwave radiation.
- Radiation and transverse kick peak at resonance ($T=T_p$).
- SC wake treat differently trailing bunches.
- Wake has an impact also in case of an array of e-Clouds.

"Vacuum Cleaner"

Not in scale in order to emphasize the concept.

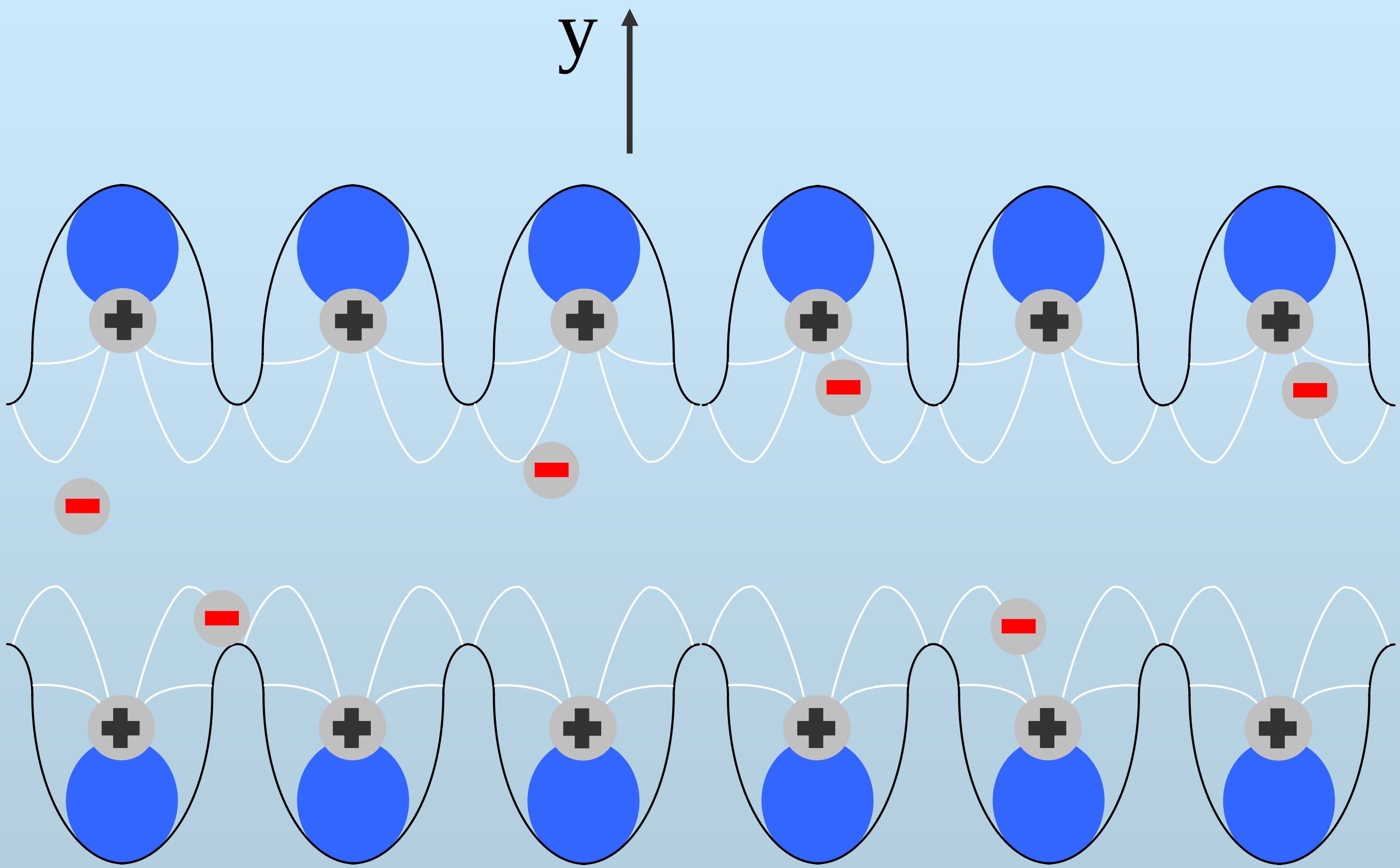


Photo-Electrons –Preliminary Conclusions

	$E_{th}[eV]$	$E_k = 2[GeV]$		$E_k = 5[GeV]$	
		$\rho_{eff}(E) = 0$	$\rho_{eff}(E) \neq 0$	$\rho_{eff}(E) = 0$	$\rho_{eff}(E) \neq 0$
N_{pe} / N_e	0	25.8	20.9	32.9	27.6
	300	0.047(0.18%)	0.047(0.22%)	1.1(3.3%)	1.1(4%)
V_{pe}	0	354.4[V]	332.9[V]	1.64[kV]	1.61[kV]
	300	19.8[V](5.6%)	19.8[V](6%)	0.97[kV](59%)	0.97[kV](60%)
N_{ph} / N_e	0	259.4		648.5	
	300	13.8(5%)		314 (48%)	
V_{ph}	0	177[kV]		0.690[MV]	
	300	6.6[kV] (4%)		0.667[MV](97%)	

- *Number of PE is one order of magnitude smaller than the photons*
- *Effective voltage associated with PE two orders of magnitude smaller*